ISSN 1330–0016
CODEN FIZBE7

# CONSTANT CUT-OFF APPROACH TO THE SKYRMION FLUID

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Received 15 May 1999; revised manuscript received 17 October 1999
Accepted 22 November 1999

We review a quantum stabilization method for the SU(2) $\sigma$-model, based on the constant cut-off limit of the cut-off quantization method developed by Balakrishna et al., which avoids the difficulties with the usual soliton boundary-conditions pointed out by Iwasaki and Ohyama. We investigate the baryon number $B = 1$ sector of the model and show that after the collective coordinate quantization, it admits a stable soliton solution which depends on a one-dimensional arbitrary constant. We then study a soliton fluid coupled to the dilaton field and the $\omega$ meson field in this model. We use the mean-field theory in which the dilaton and the $\omega$ field acquire a mean value determined by the solitons. Thus, we calculate the soliton binding energy, effective soliton mass, pressure, chemical potential and the entropy per soliton, showing that there is a good qualitative agreement of the present results with those obtained using the complete Skyrme model.

PACS numbers: 12.40 UDC 539.126

Keywords: SU(2) $\sigma$-model, constant cut-off limit of the cut-off quantization method, $B = 1$ sector, soliton fluid coupled to dilaton field, soliton binding energy, effective soliton mass, pressure, chemical potential and entropy

## 1. *Introduction*

It was shown by Skyrme [1] that baryons can be treated as solitons of a nonlinear chiral theory. The original Lagrangian of the chiral SU(2) $\sigma$-model is given by

$$L = \frac{F_\pi^2}{16} \ \mathrm{Tr} \ \partial_\mu U \partial^\mu U^+ \,, \tag{1}$$

where

$$U = \frac{2}{F_\pi}(\sigma + \mathrm{i}\,\vec{\tau}\cdot\vec{\pi}) \tag{2}$$

is a unitary operator ($UU^+ = 1$) and $F_\pi$ is the pion-decay constant. In (2) $\sigma = \sigma(\vec{r})$ is a scalar meson field and $\vec{\pi} = \vec{\pi}(\vec{r})$ is the pion-isotriplet.

The classical stability of the soliton solution to the chiral $\sigma$-model Lagrangian requires the additional ad-hoc term, proposed by Skyrme [1], to be added to (1)

$$L_{sk} = \frac{1}{32e^2}\mathrm{Tr}\,[\,U^+\partial_\mu U,\, U^+\partial_\nu U\,]^2 \tag{3}$$

with a dimensionless parameter $e$ and where $[A, B] = AB - BA$. It was shown by several authors [2] that, after the collective coordinate quantization using the spherically symmetric ansatz

$$U_0 = \exp[\mathrm{i}\,\vec{\tau}\cdot\vec{r}_0 F(r)]\,, \qquad \vec{r}_0 = \vec{r}/r\,, \tag{4}$$

the chiral model, with both (1) and (3) included, gives a good agreement with the experiment for several important physical quantities. However, the introduction of the Skyrme stabilizing term makes the analytical structure of the results complicated and in many cases difficult to handle.

Mignaco and Wulck (MW) [3] indicated, therefore, a possibility to build a stable single baryon ($n = 1$) quantum state in the simple chiral theory, with Skyrme stabilizing term (3) omitted. MW have shown that the chiral angle $F(r)$ is indeed a function of a dimensionless variable $s = \frac{1}{2}\chi"(0)r$ , where $\chi"(0)$ is an arbitrary dimensional parameter intimately connected to the usual stability argument against the soliton solution for the non-linear $\sigma$-model Lagrangian.

Using the adiabatically rotated ansatz $U(\vec{r},t) = A(t)U_0(\vec{r})A^+(t)$, where $U_0(\vec{r})$ is given by (4), MW obtained the total energy of the non-linear $\sigma$-model soliton in the form

$$E = \frac{\pi}{4}F_\pi^2\frac{1}{\chi"(0)}a + \frac{1}{2}\frac{[\chi"(0)]^3}{\frac{\pi}{4}F_\pi^2 b}J(J+1)\,, \tag{5}$$

where

$$a = \int\limits_0^\infty \left[\,\frac{1}{4}s^2\left(\frac{\mathrm{d}\mathcal{F}}{\mathrm{d}s}\right)^2 + 8\sin^2\left(\frac{1}{4}\mathcal{F}\right)\right]\mathrm{d}s \tag{6}$$

$$b = \int\limits_0^\infty \mathrm{d}s\,\frac{64}{3}s^2\sin^2\left(\frac{1}{4}\mathcal{F}\right)\,, \tag{7}$$

and $\mathcal{F}$(s) is defined by

$$F(r) = F(s) = -n\pi + \frac{1}{4}\,\mathcal{F}(s)\,. \tag{8}$$

The stable minimum of the function (5), with respect to the arbitrary dimensional scale parameter $\chi"(0)$, is

$$E = \frac{4}{3} F_\pi \left[ \frac{3}{2} \left(\frac{\pi}{4}\right)^2 \frac{a^3}{b} J(J+1) \right]^{\frac{1}{4}} . \tag{9}$$

Despite the non-existence of the stable classical soliton solution to the non-linear $\sigma$-model, it is possible, after the collective coordinate quantization, to build a stable chiral soliton at the quantum level, provided that there is a solution $F = F(r)$ which satisfies the soliton boundary conditions, i.e., $F(0) = -n\pi$ , $F(\infty) = 0$ , such that the integrals (6) and (7) exist.

However, as pointed out by Iwasaki and Ohyama [4], the quantum stabilization method in the form proposed by MW [3] is not correct since in the simple $\sigma$-model, the conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. If the condition $F(0) = -\pi$ is satisfied, Iwasaki and Ohyama obtained numerically $F(\infty) = -\pi/2$, and the chiral phase $F = F(r)$ with correct boundary conditions does not exist.

In Ref. [5], the present author suggested a method to resolve this difficulty by introducing a radial modification phase $\varphi = \varphi(r)$ in the Ansatz (4), as follows

$$U(\vec{r}) = \exp[\mathrm{i}\vec{\tau} \cdot \vec{r_0} F(r) + \mathrm{i}\varphi(r)] \,. \tag{10}$$

Such a method provides a stable chiral quantum soliton, but the resulting model is an entirely non-covariant chiral model, different from the original chiral $\sigma$-model.

In the present paper, we use the constant cut-off limit of the cut-off quantization method developed by Balakrishna, Sanyuk, Schechter and Subbaraman [6] to construct a stable chiral quantum soliton within the original chiral $\sigma$-model. We then study a soliton fluid coupled to the dilaton field and the $\omega$ meson field in this model. We use the mean field theory in which the dilaton and the $\omega$ field acquire a mean value determined by the solitons. Thus, we calculate the soliton binding energy, effective soliton mass, pressure, chemical potential and entropy per soliton, showing that there is a good qualitative agreement of the present results with those obtained using the complete Skyrme model [7].

The reason why the cut-off–approach to the problem of chiral quantum soliton works is related to the fact that the solution $F = F(r)$, which satisfies the boundary condition $F(\infty) = 0$, is singular at $r = 0$.

From the physical point of view, the chiral quantum model is not applicable to the region about the origin, since in the physical world in that region there is a quark-dominated 'bag' of the soliton. However, in the constant cut-off approach employed here, the 'cavity' in the middle of the soliton is not assumed to carry any quark degrees of freedom.

The present model differs, therefore, from the hybrid models [8], where in the Callan-Hornbostel-Klebanov (CHK) bound-state SU(3)-soliton model, a cavity populated with quarks is introduced in the centre of the soliton. The present model is fully analogous to the original Skyrme model, and our soliton is a topological soliton with the winding number equal to the baryon number. The total baryon number is determined by the soliton degrees of freedom from the region where $r$ is larger than the cut-off $\epsilon$, and there are no contributions from any quark degrees of freedom in the 'bag'. Thus, in the constant cut-off model, there is no problem with the balance of the baryon number of hyperons.

However, as argued in Ref. [6], when a cut-off $\epsilon$ is introduced, then the boundary conditions $F(\epsilon) = -n\pi$ and $F(\infty) = 0$ can be satisfied. In Ref. [6], an interesting analogy with the damped pendulum has been discussed, showing clearly that as long as $\epsilon > 0$, there is a chiral phase $F = F(r)$ satisfying the above boundary conditions. The asymptotic forms of such a solution are given by Eq. (2.2) in Ref. [6]. From these asymptotic solutions, we immediately see that for $\epsilon \to 0$, the chiral phase diverges at the lower limit.

Different applications of the constant cut-off approach have been discussed in Ref. [9].

## 2. *Constant cut-off stabilization*

The chiral soliton with baryon number $n = 1$ is given by (4), where $F = F(r)$ is the radial chiral phase function satisfying the boundary conditions $F(0) = -\pi$ and $F(\infty) = 0$.

Substituting (4) into (1), we obtain the static energy of the chiral baryon

$$M = \frac{\pi}{2} F_\pi^2 \int_{\epsilon(t)}^{\infty} \mathrm{d}r \left[ r^2 \left( \frac{dF}{dr} \right)^2 + 2\sin^2 F \right] . \tag{11}$$

In (11), we avoid the singularity of the profile function $F = F(r)$ at the origin by introducing the cut-off $\epsilon(t)$ at the lower boundary of the space interval $r \in [0, \infty]$, i.e. by working with the interval $r \in [\epsilon, \infty]$. The cut-off itself is introduced following Ref. [6] as a dynamic time-dependent variable.

From (11), we obtain the following differential equation for the profile function $F = F(r)$

$$\frac{\mathrm{d}}{\mathrm{d}r}(r^2 \frac{\mathrm{d}F}{\mathrm{d}r}) = \sin(2F) , \tag{12}$$

with the boundary conditions $F(\epsilon) = -\pi$ and $F(\infty) = 0$, such that the correct soliton number is obtained. The profile function $F = F[r; \epsilon(t)]$ now depends implicitly on time $t$ through $\epsilon(t)$. Thus, in the nonlinear $\sigma$-model Lagrangian

$$L = \frac{F_\pi^2}{16} \int \mathrm{d}^3 x \, \mathrm{Tr}(\partial_\mu U \partial^\mu U^+) , \tag{13}$$

we use the Ansätze

$$U(\vec{r},t) = A(t)U_0(\vec{r},t)A^+(t) \quad U^+(\vec{r},t) = A(t)U_0^+(\vec{r},t)A^+(t)\,, \tag{14}$$

where

$$U_0(\vec{r},t) = \exp[\mathrm{i}\,\vec{\tau}\cdot\vec{r}_0 F(r;\epsilon(t))]. \tag{15}$$

The static part of the Lagrangian (13), i.e.,

$$L = \frac{F_\pi^2}{16}\int \mathrm{d}^3x\,\mathrm{Tr}(\vec{\nabla}U\cdot\vec{\nabla}U^+) = -M\,, \tag{16}$$

is equal to minus the energy $M$ given by (11). The kinetic part of the Lagrangian is obtained using (14) with (15) and it is equal to

$$L = \frac{F_\pi^2}{16}\int \mathrm{d}^3x\,\mathrm{Tr}(\partial_0 U\partial_0 U^+) = bx^2\mathrm{Tr}(\partial_0 A\partial_0 A^+) + c[\dot{x}(t)]^2\,, \tag{17}$$

where

$$b = \frac{2\pi}{3}F_\pi^2\int_1^\infty \mathrm{d}y\,y^2\sin^2 F\,, \quad c = \frac{2\pi}{9}F_\pi^2\int_1^\infty \mathrm{d}y\,y^2\left(\frac{\mathrm{d}F}{\mathrm{d}y}\right)^2 y^2\,, \tag{18}$$

with $x(t) = [\epsilon(t)]^{3/2}$ and $y = r/\epsilon$. On the other hand, the static energy functional (11) can be rewritten as

$$M = ax^{2/3}\,, \quad a = \frac{\pi}{2}F_\pi^2\int_1^\infty \mathrm{d}y\left[y^2\left(\frac{dF}{dy}\right)^2 + 2\sin^2 F\right]. \tag{19}$$

Thus the total Lagrangian of the rotating soliton is given by

$$L = c\dot{x}^2 - ax^{2/3} + 2bx^2\dot{\alpha}_\nu\dot{\alpha}^\nu\,, \tag{20}$$

where $\mathrm{Tr}(\partial_0 A\partial_0 A^+) = 2\dot{\alpha}_\nu\dot{\alpha}^\nu$ and $\alpha_\nu$ $(\nu = 0,1,2,3)$ are the collective coordinates defined as in Ref. [10]. In the limit of a time-independent cut-off ($\dot{x}\to 0$), we can write

$$H = \frac{\partial L}{\partial\dot{\alpha}^\nu}\dot{\alpha}^\nu - L = ax^{2/3} + 2bx^2\dot{\alpha}_\nu\dot{\alpha}^\nu = ax^{2/3} + \frac{1}{2bx^2}J(J+1)\,, \tag{21}$$

where $\vec{J}^2 = J(J+1)$ is the eigenvalue of the square of the soliton laboratory angular momentum. A minimum of (21) with respect to the parameter $x$ is reached at

$$x = \left[\frac{2}{3}\frac{ab}{J(J+1)}\right]^{-3/8} \quad\Rightarrow\quad \epsilon^{-1} = \left[\frac{2}{3}\frac{ab}{J(J+1)}\right]^{1/4}. \tag{22}$$

The energy obtained by substituting (22) into (21) is given by

$$E = \frac{4}{3}\left[\frac{3}{2}\frac{a^3}{b}J(J+1)\right]^{1/4} . \tag{23}$$

This result is identical to the result obtained by Mignaco and Wulck [3], which is easily seen if we rescale the integrals $a$ and $b$ in such a way that $a \to \frac{\pi}{4}F_\pi^2 a$, $b \to \frac{\pi}{4}F_\pi^2 b$ and introduce $f_\pi = 2^{-2/3}F_\pi$. However, in the present approach, as shown in Ref. [6], there is a profile function $F = F(y)$ with the proper soliton boundary conditions $F(1) = -\pi$ and $F(\infty) = 0$ and the integrals $a$, $b$ and $c$ in (18) and (19) exist and are shown in Ref. [6] to be $a = 0.78$ GeV$^2$, $b = 0.91$ GeV$^2$ and $c = 1.46$ GeV$^2$ for $F_\pi = 186$ MeV.

Using (23), we obtain the same prediction for the mass ratio of the lowest states as Mignaco and Wulck [3] which agrees rather well with the empirical mass ratio for the $\Delta$-resonance and the nucleon. Furthermore, using the calculated values for the integrals $a$ and $b$, we obtain the nucleon mass $M(\mathrm{N}) = 1167$ MeV which is about 25% higher than the empirical value of 939 MeV. However, if we choose the pion decay constant equal to $F_\pi = 150$ MeV, we obtain $a = 0.507$ GeV$^2$ and $b = 0.592$ GeV$^2$ giving the exact agreement with the empirical nucleon mass.

Finally, it is of interest to know how large the constant cut-offs are for the above values of the pion-decay constant in order to check if they are in the physically acceptable ball park. Using (22), it is easily shown that for the nucleons ($J = \frac{1}{2}$), the cut-offs are equal to

$$\epsilon = \begin{cases} 0.22 \text{ fm} , & \text{for } F_\pi = 186 \text{ MeV} \\ 0.27 \text{ fm} , & \text{for } F_\pi = 150 \text{ MeV} \end{cases} . \tag{24}$$

Clearly, the cut-offs have to be smaller than the nucleon size (0,72 fm), and from (24), we see that this is the case. It should, however, be noted that the simple Skyrme model discussed here is at variance with some physical constraints since the isoscalar charge radius ($\approx 0.8$ fm) is identical to the baryon charge radius ($\approx 0.5$ fm).

## 3. *Dilute skyrmion fluid*

Following Ref. [7], where the skyrmion fluid was treated using the complete Skyrme model, we write the Lagrangian of the field theory, including solitons, the dilaton $\sigma$ and the $\omega$ meson, as follows

$$\begin{aligned} L &= L_{2\text{dilaton}} + L_2 - V_{\text{interaction}} - V(\sigma) + L_\omega \\ &= \mathrm{e}^{2\sigma}[\frac{1}{2}\Gamma_0^2\partial_\mu\sigma\partial^\mu\sigma - \frac{F_\pi^2}{16}\mathrm{Tr}(U^+\partial_\mu U U^+\partial^\mu U)] \\ &- g_v\omega_\mu B^\mu - B[1+\mathrm{e}^{4\sigma}(4\sigma-1)] - \frac{1}{4}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)^2 + \frac{1}{2}\mathrm{e}^{2\sigma}m_\omega^2\omega_\mu^2 . \end{aligned} \tag{25}$$

The scale invariance of the Lagrangian (25) is broken only by the anomaly, and all other terms are scale invariant, including the $\omega$ mass term with appropriate factors of $\mathrm{e}^{\sigma}$. The Lagrangian (25) possesses both isospin and chiral symmetry, since the chiral symmetry breaking pion mass term is omitted as relatively small.

The dilute skyrmion fluid, consisting of $N$ skyrmions, is built using the product Ansatz for skyrmions

$$U_N(\vec{r}, \vec{R}_1, \vec{R}_2, ..., \vec{R}_N) = U(\vec{r} - \vec{R}_1)U(\vec{r} - \vec{R}_2) \cdots U(\vec{r} - \vec{R}_N) \tag{26}$$

and the additive Ansatz for the scalar fields

$$\sigma_N = \sigma_1 + \sigma_2 + ... + \sigma_N = \sigma_0 + \delta\sigma_1 + \delta\sigma_2 + ... + \delta\sigma_N \tag{27}$$

$$\omega_N = \omega_1 + \omega_2 + ... + \omega_N = \omega_0 + \delta\omega_1 + \delta\omega_2 + ... + \delta\omega_N \tag{28}$$

where $\sigma_0$ and $\omega_0$ are the mean-field-constant values and $\delta\sigma_j$ and $\delta\omega_j$ $(1 \le j \le N)$ are the field fluctuations. The fields in (27) and (28) depend on the same arguments as the Skyrmion field they are attached to.

Due to the Ansätze, we first focus on the single baryon case where $U(\vec{r})$ is given by (4), and

$$\omega^{\mu}(\vec{r}) = [\omega(r), 0, 0, 0]\,. \tag{29}$$

Using (4) and (29) in (25), we obtain the static Skyrmion mass in the form

$$M = 4\pi \int_{\epsilon}^{\infty} r^2 \mathrm{d}r M(r) \tag{30}$$

where

$$M = \mathrm{e}^{2\sigma} \frac{F_\pi^2}{8} \left[ \left( \frac{\mathrm{d}F}{\mathrm{d}r} \right)^2 + 2\frac{\sin^2 F}{r^2} \right] + V(\sigma)$$

$$+ \mathrm{e}^{2\sigma} \frac{1}{2} \Gamma_0^2 \left( \frac{\mathrm{d}\sigma}{\mathrm{d}r} \right)^2 - \frac{1}{2} \left( \frac{\mathrm{d}\omega}{\mathrm{d}r} \right)^2 + \frac{g_V \omega \sin^2 F}{2\pi^2 r^2} \frac{\mathrm{d}F}{\mathrm{d}r} - \frac{1}{2} m_\omega^2 \omega^2 \mathrm{e}^{2\sigma}\,. \tag{31}$$

The field equations for the static profile and the meson fields are then given by

$$\mathrm{e}^{2\sigma} \left[ \frac{1}{r^2} \frac{\mathrm{d}}{\mathrm{dr}} \left( r^2 \frac{\mathrm{d}F}{dr} \right) + 2\frac{\mathrm{d}\sigma}{\mathrm{d}r}\frac{\mathrm{d}F}{\mathrm{d}r} - \frac{\sin 2F}{r^2} \right] + \frac{2g_V \sin^2 F}{\pi^2 F_\pi^2 r^2} \frac{\mathrm{d}\omega}{\mathrm{d}r} = 0 \tag{32}$$

$$\Gamma_0^2 e^{2\sigma}\left[\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}\sigma}{\mathrm{d}r}\right)+2\left(\frac{\mathrm{d}\sigma}{\mathrm{d}r}\right)^2\right]$$

$$-\frac{F_\pi^2 e^{2\sigma}}{4}\left[\left(\frac{\mathrm{d}F}{\mathrm{d}r}\right)^2+\frac{2\sin^2 F}{r^2}\right]-\frac{\mathrm{d}V_\sigma}{\mathrm{d}\sigma}+m_\omega^2\omega^2 e^{2\sigma}=0\,, \tag{33}$$

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}\omega}{\mathrm{d}r}\right)-m_\omega^2\omega e^{2\sigma}+\frac{g_V\sin^2 F}{2\pi^2 r^2}\frac{\mathrm{d}F}{\mathrm{d}r}=0\,. \tag{34}$$

Since the fluctuations of $\sigma$ and $\omega$ vanish in the mean field state, using (27) and neglecting the small contributions coming from the potential $V_\sigma$, we find that the above equations are modified by the simple scaling laws

$$r\to e^{-\sigma_0}r\,,\qquad \omega\to e^{\sigma_0}\omega\,. \tag{35}$$

It is now sufficient to solve these equations for a free skyrmion and then rescale the $\omega$ field and radial distance. The static mass of Eq. (30) is rescaled as follows [7]

$$M=e^{\sigma_0}M_0 \tag{36}$$

where $M_0$ is the mass for $\sigma=0$ .

Applying now the Lorentz boosts, following Ref. [7], to the collective coordinates $R(t)$ of each skyrmion and calculating the Hamiltonian, we obtain for the energy of a skyrmion in motion

$$E_p=(E_2+E_\sigma-E_\omega)\frac{2p^2+3M^2}{3\varepsilon M}+\frac{E}{\varepsilon}(U_\sigma-U_\omega+U_{\mathrm{int}}) \tag{37}$$

where

$$\varepsilon=(p^2+M^2)^{1/2}\,,\qquad p=\frac{Mv}{(1-v^2)^{1/2}} \tag{38}$$

and $M$ is the static mass of Eq. (30) for nonvanishing $\sigma_0$ and $\omega_0$. In (37), the following definitions have been introduced [7]

$$E_2=\frac{4\pi F_\pi^2}{8}\int_\epsilon^\infty r^2\mathrm{d}r\, e^{2\sigma}\left[\left(\frac{\mathrm{d}F}{\mathrm{d}r}\right)^2+\frac{2\sin^2 F}{r^2}\right] \tag{39}$$

$$E_\sigma=\frac{4\pi\Gamma_0^2}{2}\int_\epsilon^\infty r^2\mathrm{d}r\, e^{2\sigma}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}r}\right)^2 \tag{40}$$

$$E_\omega = 4\pi \int_\epsilon^\infty r^2 \mathrm{d}r \left(\frac{\mathrm{d}\omega}{\mathrm{d}r}\right)^2 \tag{41}$$

$$U_\sigma = 4\pi \int_\epsilon^\infty r^2 \mathrm{d}r V_\sigma \tag{42}$$

$$U_\omega = 4\pi \int_\epsilon^\infty r^2 \mathrm{d}r \mathrm{e}^{2\sigma} \frac{m_\omega^2 \omega^2}{2} \tag{43}$$

$$E_{\mathrm{int}} = \frac{2g_V}{\pi} \int_\epsilon^\infty \mathrm{d}r \omega \frac{\mathrm{d}F}{\mathrm{d}r} \sin^2 F\,, \tag{44}$$

where $\epsilon$ is the constant cut-off defined as in (22).

The energy $E_p$ defined by (37) enters the single particle distribution functions

$$n_p = \frac{1}{\exp\left(\dfrac{E_p + g_V \omega_0 - \mu}{kT}\right) + 1} \tag{45}$$

$$\overline{n_p} = \frac{1}{\exp\left(\dfrac{E_p - g_V \omega_0 + \mu}{kT}\right) + 1}\,. \tag{46}$$

The energy of $N$ skyrmions per unit volume in the mean-field approximation for symmetric nuclear matter is then given by

$$E_V = 4 \int \frac{\mathrm{d}^3 p}{(2\pi)^3} E_p (n_p + \overline{n_p}) + V_\sigma(\sigma_0) - \frac{1}{2} \mathrm{e}^{2\sigma_0} m_\omega^2 \omega_0^2 + 4 g_V \omega_0 \int \frac{\mathrm{d}^3 p}{(2\pi)^3} (n_p - \overline{n_p})\,. \tag{47}$$

At zero temperature ($T = 0$), we have $n_p = \Theta(p_F - p)$ and $\overline{n_p} = 0$ such that the equations of motion for the mean fields become

$$0 = \frac{\partial E_V}{\partial \sigma_0} = 4 \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{\partial E_p}{\partial \sigma_0} + \frac{\mathrm{d}V_{sigma}}{\mathrm{d}\sigma_0} - m_\omega^2 \mathrm{e}^{2\sigma_0} \omega_0^2 \tag{48}$$

$$0 = \frac{\partial E_V}{\partial \omega_0} = m_\omega^2 \mathrm{e}^{2\sigma_0} \omega_0 - \frac{2 g_V p_F^3}{3\pi^2}\,. \tag{49}$$

At finite temperature, instead of energy, we consider the contribution to the pressure by a single Skyrmion, given by

$$P_p = (E_2 + E_\sigma - E_\omega)\frac{2p^2 - 3M^2}{9\epsilon M} - \frac{M}{\epsilon}(U_\sigma - U_\omega) \tag{50}$$

so that the pressure per unit volume is given by

$$P_V = 4\int \frac{\mathrm{d}^3 p}{(2\pi)^3} P_p(n_p + \overline{n_p}) + V_\sigma(\sigma_0) - \frac{1}{2}\mathrm{e}^{2\sigma_0} m_\omega^2 \omega_0^2 \,. \tag{51}$$

Following Ref. [7] and using the virial theorem to the soliton profile in the case of vanishing dilaton and $\omega$ fluctuation, $E_2 = \frac{1}{2}M$, we have

$$M = \pi F_\pi^2 \mathrm{e}^{2\sigma_0} \int\limits_\epsilon^\infty r^2 \mathrm{d}r \left( \left(\frac{\mathrm{d}F}{\mathrm{d}r}\right)^2 + \frac{2\sin^2 F}{r^2} \right) = \mathrm{e}^{\sigma_0} M_0 \tag{52}$$

$$\epsilon = E_p = \sqrt{P^2 + \mathrm{e}^{2\sigma_0} M_0^2}\,, \qquad P_p = \frac{P^2}{3E_p} \tag{53}$$

where $\epsilon$ is the constant cut-off defined as in (22).

Thus, in the present approximation, the mean-field Skyrmion fluid is described in the same way as in the Dirac mean-field approach, with the additional advantage of knowing how to calculate the reaction of the single Skyrmion to the bath using dilaton scaling properties [7].

In (25), the conventional glue potential with the "bag-constant" $B \approx (240\mathrm{MeV})^4$ has been introduced. This potential reflects the trace anomaly [7], and it can be supplemented by other terms consistent with the anomaly in order to fit the properties of nuclear matter. The most important reason for adding other terms is fitting the low value of the nuclear compressibility modulus $\kappa \approx 270$ MeV. The actual choice of the functional form of these terms turns out to be relatively unimportant [7]. The constraints on $V_\sigma$, demanded by the nuclear matter phenomenology, are such that at the saturation density of nuclear matter, $\rho_0 = 0.154$ baryons/fm$^3$, we have:

a) the binding energy per nucleon is 16 MeV,

b) the binding energy has a maximum,

c) the compressibility modulus is of the order of 270 MeV and

d) the dilaton and $\omega$ fields satisfy the mean-field equations.

These conditions give the following equations

$$\omega_0 = \frac{g_V \rho_0 \mathrm{e}^{-2\sigma_0}}{m_\omega^2}\,, \qquad \frac{\partial V_\sigma}{\partial \sigma_0} - \omega_0^2 m_\omega^2 \mathrm{e}^{2\sigma_0} + Q = 0 \tag{54}$$

with

$$Q = \frac{M^2(\frac{3}{2}\pi^2\rho)^{2/3}}{\pi^2}\times$$

$$\left[\sqrt{1+M^2\left(\frac{3}{2}\pi^2\rho\right)^{-2/3}} - M^2\left(\frac{3}{2}\pi^2\rho\right)^{-2/3}\mathrm{arccoth}\sqrt{1+M^2\left(\frac{3}{2}\pi^2\rho\right)^{-2/3}}\right] \quad (55)$$

$$E_V = V_\sigma + \frac{m_\omega^2\omega_0^2}{2} + \frac{Q}{4} + \frac{\frac{3}{2}\pi^2\rho\sqrt{(\frac{3}{2}\pi^2\rho)^{2/3}+M^2}}{2\pi^2} = \rho_0(M_0 - 16\ \mathrm{MeV}) \quad (56)$$

$$\frac{E_V}{\rho_0} = \sqrt{\left(\frac{3}{2}\pi^2\rho\right)^{2/3}+M^2} + \frac{g_V^2\rho_0\mathrm{e}^{2\sigma_0}}{m_\omega^2} \quad (57)$$

$$\kappa = 9\rho_0\left[\frac{\partial^2 E_V}{\partial\rho^2} - \left(\frac{\partial^2 E_V}{\partial\rho\partial\sigma}\right)^2\left(\frac{\partial^2 E_V}{\partial\sigma^2}\right)^{-1}\right] \quad (58)$$

where

$$\frac{\partial^2 E_V}{\partial\rho^2} = \frac{(\frac{3}{2}\pi^2\rho)^{2/3}}{3\rho_0\sqrt{(\frac{3}{2}\pi^2\rho)^{2/3}+M^2}} + \frac{g_V^2\mathrm{e}^{-2\sigma_0}}{m_\omega^2} \quad (59)$$

$$\frac{\partial^2 E_V}{\partial\rho\partial\sigma} = \frac{M^2}{\sqrt{(\frac{3}{2}\pi^2\rho)^{2/3}+M^2}} - \frac{2g_V^2\rho_0\mathrm{e}^{-2\sigma_0}}{m_\omega^2} \quad (60)$$

$$\frac{\partial^2 E_V}{\partial\sigma^2} = \frac{\partial^2 V_\sigma}{\partial\sigma^2} + 4Q - \frac{3\pi^2\rho M^2}{\pi^2\sqrt{(\frac{3}{2}\pi^2\rho)^{2/3}+M^2}} + \frac{2g_V^2\rho_0^2\mathrm{e}^{-2\sigma_0}}{m_\omega^2}\,. \quad (61)$$

In order to fulfill these constraints, following Ref. [7], we introduce the aditional term to the dilaton potential which reproduces the nuclear matter phenomenology. The dilaton potential thus becomes

$$V_\sigma = B[1+\mathrm{e}^{4\sigma}(4\sigma-1)] + B[a_1(\mathrm{e}^{-\sigma}-1) + a_2(\mathrm{e}^{\sigma}-1) + a_3(\mathrm{e}^{2\sigma}-1) + a_4(\mathrm{e}^{3\sigma}-1) \quad (62)$$

where $B$ is fixed, and the anomaly condition requires $\mathrm{d}V_\sigma/\mathrm{d}\sigma = 0$ at $\sigma = 0$, which gives $a_1 = a_2 + 2a_3 + 3a_4$.

The potentials for the fluctuations of the $\sigma$ and $\omega$ fields are then determined by the averages $<\sigma>= \sigma_0$ and $<\omega>= \omega_0$ to be to the lowest order mass-term interactions

$$V(\delta\sigma) = \frac{B}{2}\delta\sigma^2\left[16\mathrm{e}^{4\sigma_0}(1+4\sigma_0) + a_1\mathrm{e}^{-\sigma_0} + a_2\mathrm{e}^{\sigma_0} + 2a_3\mathrm{e}^{2\sigma_0} + \frac{9}{2}a_4\mathrm{e}^{3\sigma_0}\right], \quad (63)$$

$$V(\delta\omega) = \frac{1}{2}\mathrm{e}^{2\sigma_0} m_\omega^2 \delta\omega^2 \,. \tag{64}$$

The effective mass of the dilaton is not fixed, because it depends on the parameter $\Gamma_0$ of Eq. (25), that has to be determined separately. It is of importance in the finite nuclei calculations only.

## 4. *Numerical results*

Using the same values of the parameters $a_1 - a_4$ as in Ref. [7], we obtain the constant cut-off results for the binding energy per nucleon in MeV as a function of $\rho/\rho_0$, given in Table 1.

*TABLE 1. Binding energy $E_B$ in* MeV *as a function of $\rho/\rho_0$.*

| $\rho/\rho_0$ | $E_B$ (MeV) | | | |
|---|---|---|---|---|
| | Normal | Abnormal | Normal Ref. [7] | Abnormal Ref. [7] |
| 0.3 | 295 | 0 | 275 | -5 |
| 0.5 | 90 | -5 | 75 | -10 |
| 1.0 | -25 | -20 | -15 | -15 |
| 1.5 | -5 | 5 | 0 | 0 |
| 2.0 | 65 | 45 | 50 | 40 |
| 2.5 | 185 | 90 | 175 | 75 |

From Table 1, we see that there is a general qualitative agreement between the present results and the results obtained using the complete Skyrme model in Ref. [7]. The more detailed discussion of the normal and abnormal solutions can be found in Ref. [7].

The constant cut-off results for the effective Skyrmion mass in MeV as a function of $\rho/\rho_0$ at four different temperatures are given in Table 2.

From Table 2, we see that there is a general qualitative agreement between the present results and the results obtained using the complete Skyrme model [7]. Similarly to the case of the complete Skyrme model, we note that in the constant cut-off approach, the nucleon mass does not decrease as in the Walecka model, but it has a minimum at about $1.5\rho/\rho_0$. A detailed discussion of the reasons for this behaviour can be found in Ref. [7].

The constant cut-off results for the pressure in MeV/fm$^3$ as a function of $\rho/\rho_0$ at four different temperatures are given in Table 3.

From Table 3, we see that there is a general qualitative agreement between the present results and the results obtained using the complete Skyrme model [7].

*TABLE 2. Effective Skyrmion Mass $M^*$ in MeV as a function of $\rho/\rho_0$.*

| $\rho/\rho_0$ | $M^*$ (MeV) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $T = 25$ MeV | | $T = 75$ MeV | | $T = 125$ MeV | | $T = 175$ MeV | |
| | | Ref. [7] | | Ref. [7] | | Ref. [7] | | Ref. [7] |
| 0 | 990 | 940 | 985 | 940 | 990 | 940 | 985 | 940 |
| 0.5 | 865 | 830 | 860 | 835 | 865 | 830 | 860 | 835 |
| 1.0 | 730 | 710 | 765 | 750 | 730 | 710 | 765 | 750 |
| 1.5 | 605 | 630 | 685 | 695 | 605 | 630 | 685 | 695 |
| 2.0 | 660 | 670 | 700 | 705 | 660 | 670 | 700 | 705 |
| 2.5 | 710 | 700 | 725 | 725 | 710 | 700 | 725 | 725 |
| 3.0 | 735 | 725 | 750 | 745 | 735 | 725 | 750 | 745 |
| 3.5 | 770 | 750 | 775 | 765 | 770 | 750 | 775 | 765 |
| 4.0 | 795 | 770 | 800 | 780 | 795 | 770 | 800 | 780 |

*TABLE 3. Pressure $P^*$ in $\mathrm{MeV/fm^3}$ as a function of $\rho/\rho_0$.*

| $\rho/\rho_0$ | $P^*$ (MeV) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $T = 25$ MeV | | $T = 75$ MeV | | $T = 125$ MeV | | $T = 175$ MeV | |
| | | Ref. [7] | | Ref. [7] | | Ref. [7] | | Ref. [7] |
| 0 | 0.01 | 0.03 | 0.04 | 0.1 | 0.1 | 0.3 | 1.5 | 4.5 |
| 0.5 | 0.3 | 0.8 | 2 | 4 | 4 | 10 | 7 | 15 |
| 1.0 | 1 | 2 | 9 | 15 | 10 | 20 | 15 | 30 |
| 1.5 | 10 | 15 | 25 | 35 | 28 | 45 | 30 | 50 |
| 2.0 | 45 | 60 | 50 | 70 | 55 | 80 | 65 | 100 |
| 2.5 | 75 | 90 | 85 | 100 | 88 | 110 | 90 | 120 |
| 3.0 | 100 | 120 | 105 | 130 | 108 | 140 | 110 | 150 |
| 3.5 | 175 | 180 | 180 | 190 | 185 | 200 | 188 | 210 |
| 4.0 | 200 | 200 | 205 | 210 | 210 | 220 | 215 | 230 |

The constant cut-off results for the chemical potential in MeV as a function of $\rho/\rho_0$ at four different temperatures are given in Table 4.

From Table 4, we see that there is a general qualitative agreement between the present results and the results obtained using the complete Skyrme model [7].

The constant cut-off results for the entropy per baryon as a function of $\rho/\rho_0$ at four different temperatures are given in Table 5.

From Table 5, we see that there is a general qualitative agreement between the present results and the results obtained using the complete Skyrme model [7].

In general all numerical results, obtained using the constant cut-off approach here, qualitatively support the results obtained in Ref. [7]. It is an expected behaviour since in the results obtained in Ref. [7], the Skyrme stabilizing term does not play a significant quantitative role. It is only needed for Skyrmion stabilization.

*TABLE 4. Chemical potential $\mu^*$ in* MeV *as a function of $\rho/\rho_0$.*

| $\rho/\rho_0$ | $\mu^*$ (MeV) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $T = 25$ MeV | | $T = 75$ MeV | | $T = 125$ MeV | | $T = 175$ MeV | |
| | | Ref. [7] | | Ref. [7] | | Ref. [7] | | Ref. [7] |
| 0 | 995 | 825 | 555 | 480 | 210 | 110 | 0 | 0 |
| 0.5 | 1115 | 890 | 815 | 740 | 670 | 540 | 260 | 280 |
| 1.0 | 1195 | 915 | 855 | 805 | 750 | 650 | 405 | 435 |
| 1.5 | 1215 | 1020 | 945 | 915 | 810 | 760 | 550 | 580 |
| 2.0 | 1255 | 1130 | 1115 | 1045 | 910 | 890 | 655 | 705 |
| 2.5 | 1305 | 1220 | 1205 | 1150 | 1005 | 1000 | 745 | 825 |
| 3.0 | 1355 | 1305 | 1255 | 1215 | 1080 | 1090 | 835 | 925 |
| 3.5 | 1395 | 1370 | 1315 | 1305 | 1155 | 1175 | 915 | 1020 |
| 4.0 | 1445 | 1435 | 1365 | 1370 | 1230 | 1250 | 995 | 1085 |

*TABLE 5. Entropy per baryon $S^*$ as a function of $\rho/\rho_0$.*

| $\rho/\rho_0$ | $S^*$ (MeV) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $T = 25$ MeV | | $T = 75$ MeV | | $T = 125$ MeV | | $T = 175$ MeV | |
| | | Ref. [7] | | Ref. [7] | | Ref. [7] | | Ref. [7] |
| 0 | 5.3 | 5.1 | 7.9 | 7.2 | 8.5 | 8.2 | 29 | 23 |
| 0.5 | 3.1 | 3 | 5.5 | 4.9 | 6.3 | 5.9 | 9.1 | 7.5 |
| 1.0 | 2.1 | 2.1 | 4.7 | 4 | 5.5 | 5 | 7.2 | 6 |
| 1.5 | 1.5 | 1.4 | 3.5 | 3 | 4.4 | 4.1 | 5.1 | 4.8 |
| 2.0 | 1.4 | 1.6 | 3.2 | 2.8 | 4.0 | 3.9 | 5.1 | 4.6 |
| 2.5 | 1.4 | 1.5 | 3.0 | 2.7 | 3.7 | 3.8 | 5.1 | 4.4 |
| 3.0 | 1.4 | 1.5 | 3.0 | 2.6 | 3.6 | 3.8 | 5.0 | 4.3 |
| 3.5 | 1.3 | 1.4 | 2.9 | 2.5 | 3.6 | 3.7 | 4.9 | 4.2 |
| 4.0 | 1.3 | 1.4 | 2.8 | 2.5 | 3.5 | 3.6 | 4.9 | 4.2 |

## 5. *Conclusions*

We have shown the possibility of using the Skyrme model for the study of a soliton fluid coupled to the dilaton field and the $\omega$ meson field, without the use of the Skyrme stabilizing term proportional to $e^{-2}$, which makes the practical calculations more complicated and introduces the problem of the choice of the stabilizing term. We used the mean-field theory in which the dilaton and the $\omega$ field acquired a mean value determined by the solitons. Thus, we have calculated the soliton binding energy, effective soliton mass, pressure, chemical potential and the entropy per soliton in the constant cut-off model.

For such a simple model with only one arbitrary dimensional constant $F_\pi$, chosen to be equal to its empirical value $F_\pi = 186$ MeV, and with the dilaton potential parameters chosen in the same way as in Ref. [7], we find that the results obtained here are in good qualitative agreement with those obtained using the complete Skyrme model [7].

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## PRISTUP SKYRMIONSKOJ TEKUĆINI STALNIM ODREZOM

Daje se pregled kvantne stabilizacijske metode za SU(2) $\sigma$-model koja se zasniva na limesu stalnog odreza kvantizacijske metode koju su razvili Balakrishna i suradnici. Ta metoda izbjegava teškoće solitonskih graničnih uvjeta kako su to uočili Iwasaki i Ohyama. Istražuje se sektor $B = 1$ modela i pokazuje kako je nakon kolektivne koordinatne kvantizacije moguće stabilno rješenje koje ovisi o samo jednoj proizvoljnoj dimenzijskoj stalnici. Zatim se proučava solitonska tekućina vezana s dilatonskim poljem i poljem $\omega$ mezona. Rabi se teorija srednjeg polja u kojoj dilaton i $\omega$ polje poprimaju srednje vrijednosti određene solitonima. Tako se izračunava energija vezanja solitona, efektivna solitonska masa, tlak, kemijski potencijal i entropija po solitonu, i pokazuje da se postiže dobro kvalitativno slaganje ovih ishoda s ishodima cjelovitog Skyrmeovog modela.