

BETHE-SALPETER AND DYSON-SCHWINGER EQUATIONS IN A WILSON  
LOOP CONTEXT IN QCD, EFFECTIVE MASS OPERATOR,  $q\bar{q}$  SPECTRUM

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We briefly discuss the quark-antiquark Bethe-Salpeter equation and the quark Dyson-Schwinger equation derived in preceding papers. We also consider the  $q\bar{q}$  quadratic mass operator  $M^2 = (w_1 + w_2)^2 + U$  obtained by a three-dimensional reduction of the BS equation and the related approximate centre-of-mass Hamiltonian or linear-mass-operator  $H_{\text{CM}} \equiv M = w_1 + w_2 + V + \dots$ . We review previous results on the spectrum and the Regge trajectories obtained by an approximate diagonalization of  $H_{\text{CM}}$  and report new results similarly obtained for the original  $M^2$ . We show that in both cases we succeed to reproduce fairly well the entire meson spectrum in the cases in which the numerical calculations were actually practicable and with the exception of the light pseudoscalar states (related to the chiral symmetry problematic). A small rearrangement of the parameters and the use of a running coupling constant is necessary in the  $M^2$  case.

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## 1. Introduction

In preceding papers [1,2] (cf. also Ref. 3), using a Feynmann-Schwinger type of path integral representation and an appropriate ansatz for the Wilson loop correlator, we have obtained a Bethe-Salpeter (BS) and a Dyson-Schwinger (DS) equation for certain “second-order” quark-antiquark and single-quark Green functions.

In principle, to solve the  $q\bar{q}$  bound state problem, one should solve the DS equation and use the resulting quark propagator in the BS equation. In practice, even an approximate treatment of such a problem in its full four-dimensional form seems to be extremely hard, and we have to resort to the use of the free propagator and of

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a three-dimensional reduction of the BS equation (instantaneous approximation). Such reduction takes the form of the eigenvalue equation for an effective squared-mass operator  $M^2 = (w_1 + w_2)^2 + U$ ,  $w_1$  and  $w_2$  being the relativistic free energies of the quarks and  $U$  an interaction related to the BS kernel.

In more conventional terms, one can also consider a centre-of-mass Hamiltonian  $H_{\text{CM}} \equiv M = w_1 + w_2 + V + \dots$ , where in the lowest order,  $V$  differs from  $U$  only for kinematic factors. This last form can be more directly compared with the usual relativistic and non-relativistic potential models and  $V$  turns out to have various significant limit expressions. In the static limit,  $V$  takes the Cornell form

$$V = -\frac{4}{3} \frac{\alpha_s}{r} + \sigma r. \quad (1)$$

In the heavy-masses limit, by an  $1/m$  expansion (and an appropriate Foldy-Wouthuysen transformation), it reproduces the semi-relativistic potential discussed in Refs. 4 and 2. If the spin-dependent terms are neglected, it becomes identical (apart from a question of ordering) to the potential corresponding to the relativistic flux-tube model [5], up to the first order in the coupling constant  $\alpha_s$  and the string tension  $\sigma$ .

In Ref. 6, we have solved numerically the eigenvalue equation for  $H_{\text{CM}}$ , as we shall explain later, neglecting the spin-orbit terms but including the hyperfine separation. We have succeeded to reproduce fairly well the entire heavy-heavy, light-light and light-heavy quarkonium spectrum and Regge trajectories when the actual calculations were feasible. The only real exception was the case of the ground light pseudoscalar mesons, for which the three-dimensional reduction of the BS equation does not seem to be appropriate, due to the chiral-symmetry-breaking problem.

Concerning the choice of the constants, the light-quark masses were fixed on typical current values,  $m_u = m_s = 10$  MeV,  $m_c = 200$  MeV and only the heavy quark masses, the strong-coupling constant and the string tension were used as fitting parameters. Good agreement with the data was found for  $m_c = 1.40$  GeV,  $m_b = 4.81$  GeV,  $\alpha_s = 0.363$  and  $\sigma = 0.175$  GeV<sup>2</sup>.

In spite of the success attained, it turns out that the quantity  $\langle V^2 \rangle$  is not negligible, bringing, e.g., to corrections ranging between few tens MeV, and 150 MeV in the  $c\bar{c}$  case. For this reason, we have repeated the calculations for the more complex operator  $M^2$ , and present the results in this paper. A good agreement is obtained again at the price of a small rearrangement of the parameters and of using a running coupling constant given by the usual perturbative expression

$$\alpha_s(\mathbf{Q}) = \frac{4\pi}{(11 - \frac{2}{3}N_f) \ln \frac{\mathbf{Q}^2}{\Lambda^2}}, \quad (2)$$

cut at a maximum value  $\alpha_s(0)$ .

We have taken  $N_f = 4$ ,  $\Lambda = 200$  MeV,  $\alpha_s(0) = 0.35$  and  $\sigma = 0.2$  GeV<sup>2</sup>, where the last two values have been chosen in order to reproduce the correct  $J/\Psi - \eta_c$  separation and the Regge trajectory slope. We have also chosen  $m_c = 1.394$  GeV

and  $m_b = 4.763$  GeV in order to reproduce exactly the masses of  $J/\Psi$  and  $\Upsilon$ ; contrary to these changes, we have left unchanged the masses of the light quarks. Our results should be compared with the classic analysis made in Ref. 9 in the framework of the relativized quark model by using a phenomenological potential depending on nine parameters and adjusted constituent masses even for the light quarks.

In Sect. 2, we review the BS and the DS equations and in Sect. 3, the three-dimensional reduction. In Sect. 4, we report and discuss the results obtained in Ref. 6 for  $H_{CM}$  and the new results for  $M^2$ .

## 2. Bethe-Salpeter and Dyson-Schwinger equations

The gauge invariant “second order” Green functions considered in Refs. 1, 2 and 3 were defined as

$$H^{\text{gi}}(x_1, x_2; y_1, y_2) = -\frac{1}{3} \text{Tr}_C \langle U(x_2, x_1) \Delta_1^\sigma(x_1, y_1; A) U(y_1, y_2) \tilde{\Delta}_2^\sigma(x_2, y_2; -\tilde{A}) \rangle, \quad (3)$$

$$H^{\text{gi}}(x - y) = i \text{Tr}_C \langle U(y, x) \Delta^\sigma(x, y; A) \rangle, \quad (4)$$

where the tilde and  $\text{Tr}_C$  denote the transposition and the trace, respectively, over the colour indices alone;  $U$  is a path-ordered gauge string joining  $a$  to  $b$  (Schwinger string),  $U(b, a) = \text{P exp} \left\{ ig \int_a^b dx^\mu A_\mu(x) \right\}$ ; while  $\Delta^\sigma(x, y; A)$  stands for the “second order” quark propagator in an external gauge field  $A^\mu$ , defined by the iterated Dirac equation

$$(D_\mu D^\mu + m^2 - \frac{1}{2} g \sigma^{\mu\nu} F_{\mu\nu}) \Delta^\sigma(x, y; A) = -\delta^4(x - y), \quad (5)$$

with  $\sigma^{\mu\nu} = (i/2)[\gamma^\mu, \gamma^\nu]$ ; finally, the angle brackets in (3) and (4) denote average over the gauge variables alone (weighted in principle with the determinant  $M_f(A)$  resulting from the explicit integration of the fermionic fields).

The quantities  $H^{\text{gi}}(x_1, x_2; y_1, y_2)$  and  $H^{\text{gi}}(x - y)$  are simply related to their ordinary “first order” counterparts and can be equivalently used for the determination of the bound state. The advantage they offer is that of admitting path-integral representations in terms of quark world lines which derive from the similar Feynman-Schwinger representation for  $\Delta^\sigma(x, y; A)$ . Such representations depend on the gauge field only through the Wilson correlators  $W[\Gamma] = \frac{1}{3} \langle \text{TrP exp} \{ ig \oint_\Gamma dx^\mu A_\mu \} \rangle$ , associated to loops  $\Gamma$  made by the quark and antiquark world lines closed by “Schwinger strings”.

In principle, as a consequence, the above correlators should determine the whole dynamics. Unfortunately, due to confinement and the consequent failure of a purely perturbative approach, a consistent analytic evaluation of  $W$  from the Lagrangian alone is not possible today, and one has to rely on models based on incomplete

theoretical arguments and lattice simulation information. The most naive, but at the same time less arbitrary assumption, consists in writing  $i \ln W$  as the sum of its perturbative expression and an area term (modified area-law (MAL) model)

$$i \ln W = i (\ln W)_{\text{pert}} + \sigma S_{\text{min}}, \quad (6)$$

where the first quantity is supposed to give correctly the short-range limit and the second the long-range one. Notice that, in principle, any more sophisticated model could be used under the condition that it preserves certain general properties of functional derivability of the exact expression. In practice, not even (6) can be treated exactly. Actually, one has to replace the minimal surface  $S_{\text{min}}$  by its "equal-time straight-line approximation", defined as the surface spanned by straight lines joining equal-time opposite points of the loop  $\Gamma$  in the  $q\bar{q}$  centre-of-mass frame.

The path integral representations obtained in such a way could be used directly for numerical calculations or for analytic developments. In the latter context, it is convenient to consider another type of second-order functions,  $H(x_1, x_2, y_1, y_2)$  and  $H(x-y)$ , obtained from  $H^{\text{gl}}(x_1, x_2, y_1, y_2)$  and  $H^{\text{gl}}(x-y)$  by omitting in their path-integral representation the contributions to  $i \ln W$ , coming from gluon lines or straight lines involving points of the Schwinger strings. In the limit of vanishing  $x_1 - x_2$ ,  $y_1 - y_2$  or  $x - y$ , such new quantities coincide with the original ones and are completely equivalent to them for what concerns the determination of the bound states, effective masses, quark condensates, etc.

By an appropriate recurrence method, an inhomogeneous BS equation and a DS equation can be derived for  $H(x_1, x_2, y_1, y_2)$  and  $H(x-y)$ , respectively. In the momentum space, the corresponding homogeneous BS equation can be written (in a  $4 \times 4$  matrix representation)

$$\begin{aligned} \Phi_P(k) = & -i \int \frac{d^4 u}{(2\pi)^4} \hat{I}_{ab} \left( k - u, \frac{1}{2}P + \frac{k+u}{2}, \frac{1}{2}P - \frac{k+u}{2} \right) \\ & \hat{H}_1 \left( \frac{1}{2}P + k \right) \sigma^a \Phi_P(u) \sigma^b \hat{H}_2 \left( -\frac{1}{2}P + k \right), \end{aligned} \quad (7)$$

where  $\Phi_P(k)$  denotes an appropriate wave function and the centre-of-mass frame has to be understood; i.e.,  $P = (m_B, \mathbf{0})$ ,  $m_B$  being the bound state mass. Similarly, in terms of the irreducible self-energy defined by  $\hat{H}(k) = \hat{H}_0(k) + i\hat{H}_0(k)\hat{\Gamma}(k)\hat{H}(k)$ , the DS equation can be written also

$$\hat{\Gamma}(k) = \int \frac{d^4 l}{(2\pi)^4} \hat{I}_{ab} \left( k - l, \frac{k+l}{2}, \frac{k+l}{2} \right) \sigma^a \hat{H}(l) \sigma^b. \quad (8)$$

Notice that, in principle, (7) and (8) are exact equations. However, the kernels  $\hat{I}_{ab}$  are generated in the form of an expansion in  $\alpha_s$  and  $\sigma$ . In the lowest order in both such constants, we have explicitly

$$\begin{aligned}
\widehat{I}_{0;0}(Q; p, p') &= 16\pi\frac{4}{3}\alpha_s p^\alpha p'^\beta \widehat{D}_{\alpha\beta}(Q) \\
&+ 4\sigma \int d^3\zeta e^{-i\mathbf{Q}\cdot\zeta} |\zeta| \epsilon(p_0) \epsilon(p'_0) \int_0^1 d\lambda \{p_0^2 p'^2 - [\lambda p'_0 \mathbf{p}_T + (1-\lambda)p_0 \mathbf{p}'_T]^2\}^{\frac{1}{2}} \\
\widehat{I}_{\mu\nu;0}(Q; p, p') &= 4\pi i \frac{4}{3} \alpha_s (\delta_\mu^\alpha Q_\nu - \delta_\nu^\alpha Q_\mu) p'_\beta \widehat{D}_{\alpha\beta}(Q) \\
&- \sigma \int d^3\zeta e^{-i\mathbf{Q}\cdot\zeta} \epsilon(p_0) \frac{\zeta_\mu p_\nu - \zeta_\nu p_\mu}{|\zeta| \sqrt{p_0^2 - \mathbf{p}_T^2}} p'_0 \\
\widehat{I}_{0;\rho\sigma}(Q; p, p') &= -4\pi i \frac{4}{3} \alpha_s p^\alpha (\delta_\rho^\beta Q_\sigma - \delta_\sigma^\beta Q_\rho) \widehat{D}_{\alpha\beta}(Q) \\
&+ \sigma \int d^3\zeta e^{-i\mathbf{Q}\cdot\zeta} p_0 \frac{\zeta_\rho p'_\sigma - \zeta_\sigma p'_\rho}{|\zeta| \sqrt{p_0^2 - \mathbf{p}_T^2}} \epsilon(p'_0) \\
\widehat{I}_{\mu\nu;\rho\sigma}(Q; p, p') &= \pi \frac{4}{3} \alpha_s (\delta_\mu^\alpha Q_\nu - \delta_\nu^\alpha Q_\mu) (\delta_\rho^\beta Q_\sigma - \delta_\sigma^\beta Q_\rho) \widehat{D}_{\alpha\beta}(Q)
\end{aligned} \tag{9}$$

where in the second and in the third equation,  $\zeta_0 = 0$  has to be understood. Notice that the use of (2) in (9) would amount to include higher-order contributions in  $\alpha_s$ .

### 3. Three-dimensional reduction of the BS equation

To obtain from (7) a three-dimensional equation, we can perform on such equation the so-called instantaneous approximation. This consists in replacing in (7)  $\widehat{H}_2^{(j)}(p)$  with the free-quark propagator  $-i/(p^2 - m^2)$  and the kernel  $\widehat{I}_{ab}$  with  $\widehat{I}_{ab}^{\text{inst}}(\mathbf{k}, \mathbf{k}')$ , obtained from  $\widehat{I}_{ab}$  setting  $k_0 = k'_0 = (1/2)[\eta_2(w_1 + w'_1) - \eta_1(w_2 + w'_2)]$  with  $w_j = \sqrt{m_j^2 + \mathbf{k}^2}$  and  $w'_j = \sqrt{m_j^2 + \mathbf{k}'^2}$ . Then, by performing explicitly the integration over  $k'_0$  and further integrating the resulting expression in  $k_0$ , we obtain

$$(w_1 + w_2)^2 \varphi_{m_B}(\mathbf{k}) \tag{10}$$

$$+ \int \frac{d^3k'}{(2\pi)^3} \sqrt{\frac{w_1 + w_2}{2w_1 w_2}} \widehat{I}_{ab}^{\text{inst}}(\mathbf{k}, \mathbf{k}') \sqrt{\frac{w'_1 + w'_2}{2w'_1 w'_2}} \sigma^a \varphi_{m_B}(\mathbf{k}') \sigma^b = m_B^2 \varphi_{m_B}(\mathbf{k}),$$

with  $\varphi_P(\mathbf{k}) = \sqrt{(2w_1 w_2)/(w_1 + w_2)} \int_{-\infty}^{\infty} dk_0 \Phi_P(k)$ .

Eq. (10) is the eigenvalue equation for the squared mass operator,

$$M^2 = M_0^2 + U \tag{11}$$

with  $M_0 = \sqrt{m_1^2 + \mathbf{k}^2} + \sqrt{m_2^2 + \mathbf{k}^2}$  and

$$\langle \mathbf{k} | U | \mathbf{k}' \rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{w_1 + w_2}{2w_1 w_2}} \widehat{I}_{ab}^{\text{inst}}(\mathbf{k}, \mathbf{k}') \sqrt{\frac{w'_1 + w'_2}{2w'_1 w'_2}} \sigma_1^a \sigma_2^b. \tag{12}$$

The quadratic form of Eq. (11) obviously derives from the second-order character of the formalism we have used.

In more usual terms, one can also write

$$H_{\text{CM}} \equiv M = M_0 + V + \dots, \quad (13)$$

with

$$\langle \mathbf{k}|V|\mathbf{k}' \rangle = \frac{1}{w_1 + w_2 + w'_1 + w'_2} \langle \mathbf{k}|U|\mathbf{k}' \rangle = \frac{1}{(2\pi)^3} \frac{1}{4\sqrt{w_1 w_2 w'_1 w'_2}} \widehat{T}_{ab}^{\text{inst}}(\mathbf{k}, \mathbf{k}') \sigma_1^a \sigma_2^b. \quad (14)$$

In (13), the dots stand for higher-order terms in  $\alpha_s$  and  $\sigma$ , and kinematic factors equal to 1 on the energy shell have been neglected.

From Eqs. (12) and (9), one obtains explicitly

$$\begin{aligned} \langle \mathbf{k}|U|\mathbf{k}' \rangle = & \sqrt{\frac{(w_1 + w_2)(w'_1 + w'_2)}{w_1 w_2 w'_1 w'_2}} \left\{ \left[ -\frac{4\alpha_s}{3} \frac{1}{\pi^2} \frac{1}{\mathbf{Q}^2} \left( q_{10} q_{20} + \mathbf{q}^2 - \frac{(\mathbf{Q} \cdot \mathbf{q})^2}{\mathbf{Q}^2} \right) \right. \right. \\ & + \frac{i}{2\mathbf{Q}^2} \mathbf{k} \times \mathbf{k}' \cdot (\sigma_1 + \sigma_2) + \frac{1}{2\mathbf{Q}^2} [q_{20}(\alpha_1 \cdot \mathbf{Q}) - q_{10}(\alpha_2 \cdot \mathbf{Q})] \\ & + \frac{1}{6} \sigma_1 \cdot \sigma_2 + \frac{1}{4} \left( \frac{1}{3} \sigma_1 \cdot \sigma_2 - \frac{(\mathbf{Q} \cdot \sigma_1)(\mathbf{Q} \cdot \sigma_2)}{\mathbf{Q}^2} \right) + \frac{1}{4\mathbf{Q}^2} (\alpha_1 \cdot \mathbf{Q})(\alpha_2 \cdot \mathbf{Q}) \left. \right] \\ & + \frac{1}{(2\pi)^3} \int d^3 \mathbf{r} e^{i\mathbf{Q} \cdot \mathbf{r}} J^{\text{inst}}(\mathbf{r}, \mathbf{q}, q_{10}, q_{20}) \left. \right\} \end{aligned} \quad (15)$$

with

$$\begin{aligned} J^{\text{inst}}(\mathbf{r}, \mathbf{q}, q_{10}, q_{20}) = & \frac{\sigma r}{q_{10} + q_{20}} [q_{20}^2 \sqrt{q_{10}^2 - \mathbf{q}_T^2} + q_{10}^2 \sqrt{q_{20}^2 - \mathbf{q}_T^2}] \\ & + \frac{q_{10}^2 q_{20}^2}{|\mathbf{q}_T|} (\arcsin \frac{|\mathbf{q}_T|}{q_{10}} + \arcsin \frac{|\mathbf{q}_T|}{q_{20}}) \\ - \frac{\sigma}{r} \left[ & \frac{q_{20}}{\sqrt{q_{10}^2 - \mathbf{q}_T^2}} (\mathbf{r} \times \mathbf{q} \cdot \sigma_1 + i q_{10} (\mathbf{r} \cdot \alpha_1)) + \frac{q_{10}}{\sqrt{q_{20}^2 - \mathbf{q}_T^2}} (\mathbf{r} \times \mathbf{q} \cdot \sigma_2 - i q_{20} (\mathbf{r} \cdot \alpha_2)) \right]. \end{aligned} \quad (16)$$

Here  $\alpha_j^k$  denote the usual Dirac matrices  $\gamma_j^0 \gamma_j^k$ ,  $\sigma_j^k$  the  $4 \times 4$  Pauli matrices  $\begin{pmatrix} \sigma_j^k & 0 \\ 0 & \sigma_j^k \end{pmatrix}$  and obviously  $\mathbf{q} = (\mathbf{k} + \mathbf{k}')/2$ ,  $\mathbf{Q} = \mathbf{k} - \mathbf{k}'$ ,  $q_{j0} = (w_j + w'_j)/2$ . Notice that, due to the terms in  $\alpha_j^k$ , such  $U$  is self-adjoint only with reference to the undefined metric operator  $\gamma_1^0 \gamma_2^0$ .

Due to (14), the potential  $V$  can be obtained from  $U$  as given by Eqs. (15) and (16), by the kinematic replacement  $\sqrt{\frac{(w_1 + w_2)(w'_1 + w'_2)}{w_1 w_2 w'_1 w'_2}} \rightarrow \frac{1}{2\sqrt{w_1 w_2 w'_1 w'_2}}$ .

#### 4. Determination of the spectrum

In Ref. 6, we have evaluated the eigenvalues of the operator  $H_{\text{CM}}$  for the potential  $V$  discussed above, omitting the spin-orbit terms and including only the hyperfine splitting. The numerical procedure we have followed is very simple. It consists in solving first the eigenvalue equation for the static potential (1) by the Rayleigh-Ritz method [7] using the three-dimensional harmonic oscillator basis, diagonalizing a  $30 \times 30$  matrix. Then, we have evaluated the quantities  $\langle \psi_\nu | H_{\text{CM}} | \psi_\nu \rangle$  for the eigenfunctions  $\psi_\nu$  obtained in the first step, choosing the scale parameter occurring in the basis in order to make minimum the ground-state mass  $\langle \psi_{1S} | H_{\text{CM}} | \psi_{1S} \rangle$ . Notice that the determination of  $\langle \psi_\nu | V | \psi_\nu \rangle$  for the exact  $V$  is not trivial, since, in general, one should evaluate five-dimensional integrals of highly singular functions. For this reason, we used two different expansions for high and low transversal momentum (angular momentum) that allows to reduce to the three-dimensional integrals and treated the singularity with the method suggested in Ref. 8.

The procedure we have followed for the determination of the eigenvalues of  $M^2$  is essentially the same. Again, we solve first the eigenvalue equation for  $H_{\text{CM}}$  with the static potential, and then we evaluate the quantities  $\langle \psi_\nu | M^2 | \psi_\nu \rangle$ . In this case, the hyperfine splitting is determined by the equation

$$({}^3M_{nl})^2 - ({}^1M_{nl})^2 = \frac{32}{9\pi} \int_0^\infty dk k^2 \int_0^\infty dk' k'^2 \Psi_{nl}^*(k) \Psi_{nl}(k') \sqrt{\frac{w_1 + w_2}{w_1 w_2}} \sqrt{\frac{w'_1 + w'_2}{w'_1 w'_2}} \int_{-1}^1 d\xi \alpha_s(\mathbf{Q}) P_l(\xi), \quad (17)$$

which is more complicated than the corresponding equation in the case of  $H_{\text{CM}}$  [6].

Both the new results based on  $M^2$  (crosses in Figs. 1, 2 and 3 and dashed lines in

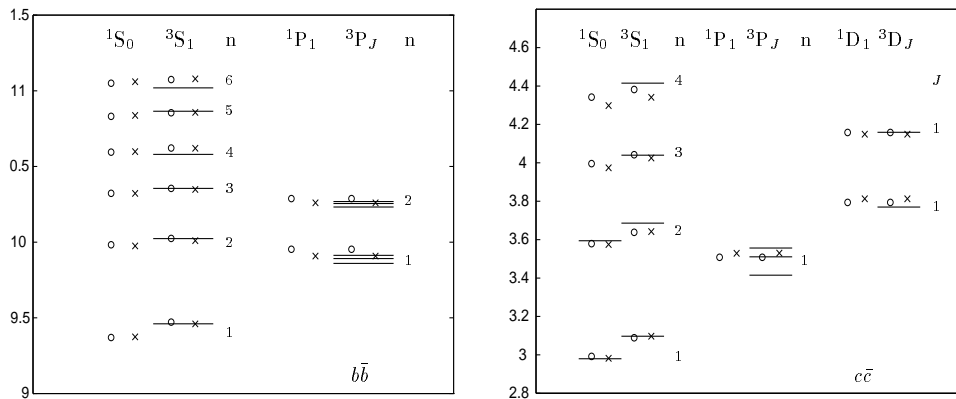


Fig. 1. Heavy-heavy quarkonium spectra.

Fig. 4) and the old ones based on  $H_{CM}$  (circlets in Figs. 1, 2 and 3 and dotted lines in Fig. 4) are reported in the figures for the parameters discussed in the Introduction.

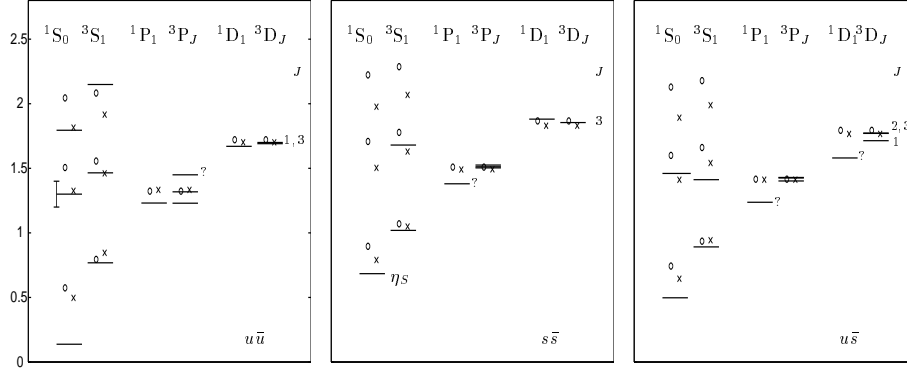


Fig. 2. Light-light quarkonium spectra. Question marks indicate states whose assignment to a multiplet is not obvious. Position of  $\eta_S$  is derived from  $\eta$  and  $\eta'$ , by standard mixture assumption.

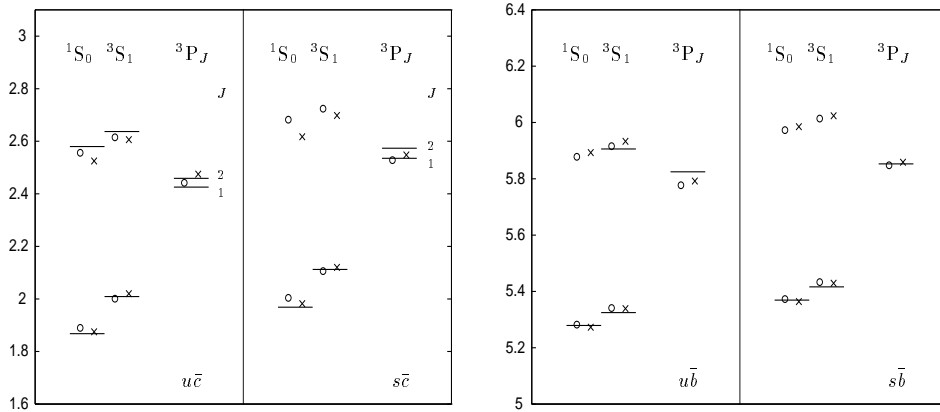


Fig. 3. Light-heavy quarkonium spectra.

For the  $l > 0$  cases, masses represent the centre-of-mass of the multiplets. In both cases, the agreement with the data is on the whole good, not only for bottomium and charmonium (as in ordinary potential models), but also the light-light and light-heavy systems. Notice that the quadratic formulation seems to give a better low angular momentum light-light spectrum, while perhaps the linear formulation gives better Regge trajectories. The Regge trajectories in the quadratic case can



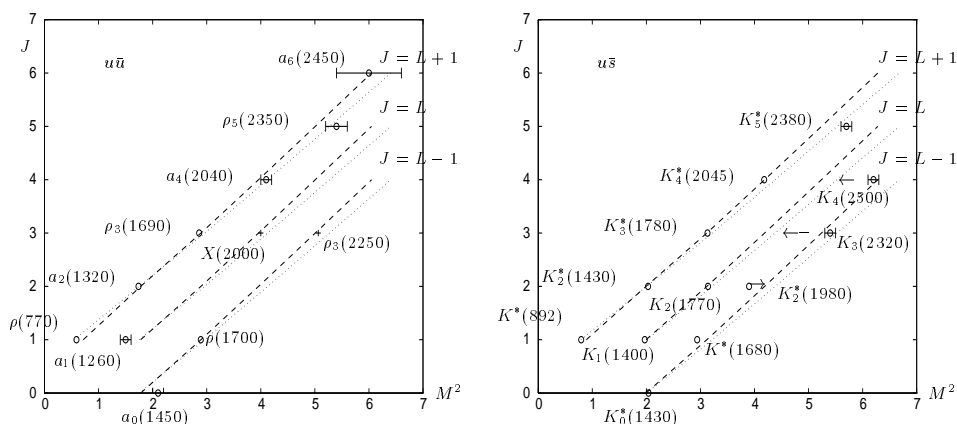


Fig. 4.  $u\bar{u}$  and  $u\bar{s}$  Regge trajectories.

be improved rising the value of  $\sigma$  at the price, however, of making more difficult the fitting of the light-heavy states (perhaps the MAL model is too naive). As mentioned, the only serious disagreement remains that of the light pseudoscalar mesons.

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BETHE-SALPETEROVA I DYSON-SCHWINGEROVA JEDNADŽBA I  
WILSONOVA PETLJA U QCD, UČINKOVIT OPERATOR MASE I  $q\bar{q}$   
SPEKTAR

Kratko se raspravlja Bethe-Salpeterova jednadžba za sustav kvark-antikvark i Dyson-Schwingerova jednadžba za kvarkove koje su ranije izvedene,  $q\bar{q}$  kvadratični operator mase  $M^2 = (w_1 + w_2)^2 + U$  koji je izveden iz BS jednadžbe te približna Hamiltonova funkcija za centar mase odnosno linearni operator mase  $H_{CM} \equiv M = w_1 + w_2 + V + \dots$ . Izlažu se raniji ishodi računa za spektar mezona i Reggeove putanje koji su dobiveni približnom dijagonalizacijom  $H_{CM}$ , te navode novi ishodi koji su slično postignuti za izvorni  $M^2$ . Pokazuje se kako se u oba slučaja prilično dobro postiže cjelokupan spektar mezona za slučajeve kada su numerički računi provedivi i uz isključenje lakih pseudoskalarnih stanja (povezanih s problematikom kiralne simetrije). U slučaju  $M^2$  potrebna su mala podešavanja parametara i jedna stalnica vezanja.