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PLANE SYMMETRIC VISCOUS FLUID UNIVERSE WITH DECAYING VACUUM ENERGY DENSITY Λ

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Plane symmetric viscous fluid cosmological models of the universe with a variable cosmological term are investigated. The viscosity coefficient of bulk viscous fluid is assumed to be a power function of mass density, whereas the coefficient of shear viscosity is taken as a constant. To get the deterministic solutions of the Einstein's field equations, the free gravitational field is assumed to be of type D which is of the next order in the hierarchy of Petrov classification. The cosmological constant Λ is found to be a decreasing function of time and positive which is corroborated by the results from recent supernovae Ia observations. The physical and geometric aspects of the models are discussed.

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1. *Introduction*

The problem of the cosmological constant is salient yet unsettled in cosmology. The smallness of the effective cosmological constant recently observed ($\Lambda_0 \leq 10^{-56}\text{cm}^{-2}$) poses the most difficult problems involving cosmology and elementary particle physics theory. To explain the striking cancellation between the "bare" cosmological constant and the ordinary vacuum energy contributions of the quantum fields, many mechanisms have been proposed [1]. The "cosmological constant problem" can be expressed as the discrepancy between the negligible value of Λ for the present universe as seen by the successes of Newton's theory of gravitation [2], whereas the values 10^{50} larger is expected by the Glashow-Salam-Weinberg model [3], and by grand unified theory (GUT) it should be 10^{107} larger [4]. The cosmological term Λ is then small at the present epoch simply because the universe is too old. The problem in this approach is to determine the right dependence of Λ upon R or t.

Models with a relic cosmological constant Λ have received ample attention among researchers recently for various reasons (see Refs. [5] – [10] and references therein). Some of the recent discussions on the cosmological constant "problem" and on cosmology with a time-varying cosmological constant by Ratra and Peebles [11], Dolgov et al. [12, 13], Dolgov [14], and Sahni and Starobinsky [15] point out that in the absence of any interaction with matter or radiation, the cosmological constant remains a "constant", however, in the presence of interactions with matter or radiation, a solution of Einstein equations and the assumed equation of covariant conservation of stress-energy with a time-varying Λ can be found. For these solutions, conservation of energy requires decrease in the energy density of the vacuum component to be compensated by a corresponding increase in the energy density of matter or radiation. Earlier researches on this topic, are contained in Zeldovich [16], Weinberg [1] and Carroll, Press and Turner [17]. Recent observations by Perlmutter et al. [18] and Riess et al. [19] strongly favour a significant and positive value of Λ. Their findings arising from the study of more than 50 type Ia supernovae with redshifts in the range $0.10 \leq z \leq 0.83$ suggest Friedmann models with negative pressure matter such as a cosmological constant (Λ), domain walls or cosmic strings (Vilenkin [20], Garnavich et al. [21]). Recently, Carmeli and Kuzmenko [22], and Behar and Carmeli [23] have shown that the cosmological relativistic theory predicts the value for cosmological constant $\Lambda = 1.934 \times 10^{-35} s^{-2}$. This value of "$\Lambda$" is in excellent agreement with the measurements recently obtained by the High-Z Supernova Team and Supernova Cosmological Project (Garnavich et al. [21], Perlmutter et al. [18], Riess et al. [19], Schmidt et al. [24]) The main conclusion of these observations is that the expansion of the universe is accelerating.

Several ansätze have been proposed in which the Λ term decays with time (see Refs. Gasperini [25], Berman [26], Berman et al. [27], Özer and Taha [7], Freese et al. [8], Peebles and Ratra [23], Chen and Hu [29], Abdussattar and Viswakarma [30], Gariel and Le Denmat [31], Pradhan [32], Pradhan et al. [33]). Of the special interest is the ansatz $\Lambda \propto S^{-2}$ (where S is the scale factor of the Robertson-Walker metric) by Chen and Wu [29], which has been considered or modified by several authors (Abdel-Rahaman [34], Carvalho et al. [9], Silveira and Waga [10], Vishwakarma [35]).

Astronomical observations of large-scale distribution of galaxies of our universe show that the distribution of matter can be satisfactorily described by a perfect fluid. However, it has been conjectured that some time during an earlier phase in the evolution of the universe when galaxies were formed, the material distribution behaved like a viscous fluid. Bulk viscosity is associated with the GUT phase transition and string creation. It is, therefore, of great interest to obtain cosmological models in such distributions. It is also well known that there is a certain degree of anisotropy in the actual universe. We, therefore, choose the metric for the cosmological model to be plane-symmetric.

Recently, Pradhan et al. [36] have investigated a new class of plane symmetric viscous fluid cosmological models of the universe with a variable cosmological constant where the coefficient of shear viscosity is taken to be proportional to the rate of expansion in the model. In the present paper, by considering the free gravitational field to be of type D, a new class of plane symmetric viscous fluid cosmological models is obtained which is of the next order in the hierarchy of Petrov classification. The paper is organized as follows. The metric and the field equations are presented in Section 2. In Section 3, the general solutions of the field equations by considering the free gravitational field to be of the Petrov type D classification is discussed. In Subsections 3.1 and 3.2, two types of models are found and their physical and geometric features are discussed. The bulk viscosity is assumed to be simple power of energy density $\xi = \xi_0 \rho^n$ and the coefficient of shear viscosity is considered to be constant η_0. In both subsections we consider the solutions for $n = 0$ and $n = 1$. Finally in Section 4 main conclusion is presented.

2. *The metric and field equations*

We consider the metric in the form of Marder [37]

$$\mathrm{d}s^2 = A^2(\mathrm{d}x^2 - \mathrm{d}t^2) + B^2\mathrm{d}y^2 + C^2\mathrm{d}z^2, \tag{1}$$

where the metric potentials A, B and C are functions of t alone. This ensures the model to be spatially homogeneous. This is a transform form of the metric of Bianchi type I spacetime in comoving coordinates which has been studied by a number of authors (e.g., Heckmann and Schucking [38], Thorne [39] and Roy and Prakash [40]).

The energy-momentum tensor for a viscous fluid distribution is given by Landau and Lifshitz [41]

$$T_i^j = (\rho + p)v_i v^j + p g_i^j - \eta(v_{i:}^j + v_{;i}^j + v^j v^l v_{i;l} + v_i v^l v_{;l}^j)$$

$$- \left(\xi - \frac{2}{3}\eta\right) v_{;l}^l (g_i^j + v_i v^j) \, . \tag{2}$$

Here ρ, p, η and ξ are energy density, isotropic pressure, the coefficient of shear Viscosity and bulk viscous coefficient respectively and v^i is the flow vector satisfying the relation

$$g_{ij} v^i v^j = -1. \tag{3}$$

The semicolon (;) indicates covariant differentiation. We choose the coordinates to be comoving, so that $v^1 = v^2 = v^3 = 0$ and $v^4 = 1/A$.

The Einstein's field equations (in gravitational units $c = 1$, $G = 1$) read as

$$R_i^j - \frac{1}{2} R g_i^j + \Lambda g_i^j = -8\pi T_i^j, \tag{4}$$

for the line element (1) has been set up as

$$\frac{1}{A^2}\left[\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}-\frac{\dot{B}\dot{C}}{BC}-\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}\right]-\Lambda(t)=8\pi\left[p-2\eta\frac{\dot{A}}{A^2}-\left(\xi-\frac{2}{3}\eta\right)v^l_{;l}\right], \quad (5)$$

$$\frac{1}{A^2}\left[\frac{\dot{A}^2}{A^2}-\frac{\ddot{A}}{A}-\frac{\ddot{C}}{C}\right]-\Lambda(t)=8\pi\left[p-2\eta\frac{\dot{B}}{AB}-\left(\xi-\frac{2}{3}\eta\right)v^l_{;l}\right], \quad (6)$$

$$\frac{1}{A^2}\left[\frac{\dot{A}^2}{A^2}-\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}\right]-\Lambda(t)=8\pi\left[p-2\eta\frac{\dot{C}}{AC}-\left(\xi-\frac{2}{3}\eta\right)v^l_{;l}\right], \quad (7)$$

$$\frac{1}{A^2}\left[\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}\right]+\Lambda(t)=8\pi\rho\,. \quad (8)$$

Here, and also in the following expressions a dot indicates ordinary differentiation with respect to t.

3. *Solutions of the field equations*

Equations (5)-(8) are four equations in eight unknowns A, B, C, p, ρ, η, ξ and Λ. Equations (5)-(8) are not independent, but are related by the contracted Bianchi identities. In the present case, they lead to the single condition

$$\frac{\mathrm{d}\rho}{\mathrm{d}t}+(p+\rho)\ln(ABC)-\left(\rho-\frac{2}{3}\eta\right)\frac{1}{A}\left(\frac{\mathrm{d}}{\mathrm{d}t}\ln(ABC)\right)^2-$$
$$\frac{2\eta}{A}\left(\frac{\dot{A}^2}{A^2}+\frac{\dot{B}^2}{B^2}+\frac{\dot{C}^2}{C^2}\right)=0\,. \quad (9)$$

For complete solutions of equations (5) – (8), we need four extra conditions. The coefficient of shear viscosity η is taken as constant η_0. The research on exact solutions is based on some physically reasonable restrictions used to simplify the Einstein equations. Although the distribution of matter at each point determines the nature of expansion in the model, the latter is also affected by the free gravitational field through its effect on the expansion, vorticity and shear in the fluid flow. A prescription of such a field may, therefore, be made on an *a priori* basis. The cosmological models of Friedmann, Robertson and Walker, as well as the universe of Einstein and de Sitter, have vanishing free gravitational fields. Here we choose the free gravitational field to be of type D which is of the next order in the hierarchy of Petrov classification. This requires that either

$$(a) \quad C^{12}{}_{12}=C^{13}{}_{13},$$

or

$$(b) \quad C^{12}{}_{12} = C^{23}{}_{23}.$$

Conditions (a) and (b) are identically satisfied if $B = C$ and $A = C$, respectively. However, we shall assume A, B, C to be unequal on account of the supposed anisotropy.

From Eqs. (5) and (6) we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{A}}{A}\right) + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) - \frac{\ddot{B}}{B} - \frac{\dot{B}\dot{C}}{BC} = 16\pi\eta_0 A\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right). \tag{10}$$

Also from Eqs. (6) and (7) we obtain

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} = 16\pi\eta_0 A\left(\frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right). \tag{11}$$

3.1. The first model

The condition

$$C^{12}{}_{12} = C^{13}{}_{13} \tag{12}$$

leads to

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + 2\frac{\dot{A}}{A}\left(\frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right) = 0\,. \tag{13}$$

Equations (11) and (13 lead to

$$A = \frac{1}{8\pi\eta_0 t + a}\,, \tag{14}$$

where a is a constant of integration. From Eqs. (13) and (14) we obtain

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} = -\frac{16\pi\eta_0}{(8\pi\eta_0 t + a)}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right), \tag{15}$$

which on integration gives

$$\dot{B}C - B\dot{C} = \frac{b}{(8\pi\eta_0 t + a)^2}\,, \tag{16}$$

where b is an integrating constant. From Eqs. (10) and (14) we get

$$\left(\frac{8\pi\eta_0}{8\pi\eta_0 t + a}\right)^2 + \frac{8\pi\eta_0}{(8\pi\eta_0 t + a)}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \frac{16\pi\eta_0}{(8\pi\eta_0 t + a)}\frac{\dot{B}}{B} + \frac{\ddot{B}}{B} + \frac{\dot{B}\dot{C}}{BC} = 0\,. \tag{17}$$

From Eqs. (16) and (17) we obtain

$$B = \frac{K(Lt+M)^{\frac{1}{2}+\frac{b}{2L}}}{(8\pi\eta_0 t + a)}\,, \tag{18}$$

and

$$C = \frac{(Lt+M)^{\frac{1}{2}-\frac{b}{2L}}}{K(8\pi\eta_0 t + a)}\,, \tag{19}$$

where K, L and M are constants of integration.

Hence, the geometry of the space time (1) takes the form

$$\mathrm{d}s^2 = \frac{1}{(8\pi\eta_0 t + a)^2}(\mathrm{d}x^2 - \mathrm{d}t^2) + \frac{K^2(Lt+M)^{1+\frac{b}{L}}}{(8\pi\eta_0 t + a)^2}\mathrm{d}y^2 + \frac{(Lt+M)^{1-\frac{b}{L}}}{K^2(8\pi\eta_0 t + a)^2}\mathrm{d}z^2. \tag{20}$$

The pressure (p) and density (ρ) for the universe (20) are given by

$$8\pi p = -192\pi^2\eta_0^2 + \frac{(L^2-b^2)(8\pi\eta_0 t + a)^2}{4(Lt+M)^2} + \frac{32\pi\eta_0 L(8\pi\eta_0 t + a)}{3(Lt+M)}$$
$$-8\pi\xi\left(24\pi\eta_0 - \frac{(8\pi\eta_0 t + a)L}{(Lt+M)}\right) - \Lambda(t)\,, \tag{21}$$

$$8\pi\rho = 192\pi^2\eta_0^2 - \frac{16\pi\eta_0 L(8\pi\eta_0 t + a)}{(Lt+M)} + \frac{(L^2-b^2)(8\pi\eta_0 t + a)^2}{4(Lt+M)^2} + \Lambda(t)\,. \tag{22}$$

For the specification of ξ, we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho\,, \tag{23}$$

where γ ($0 \le \gamma \le 1$) is constant. Thus, given $\xi(t)$ we can solve for the cosmological parameters. In most of the investigation involving bulk viscosity is assumed to be a simple power function of the energy density (Pavon [42], Maartens [43], Zimdahl [44], Santos [45])

$$\xi(t) = \xi_0\rho^n, \tag{24}$$

where ξ_0 and n are constants. For small density, n may even be equal to unity as used in Murphy's work [46] for simplicity. If $n = 1$, Eq. (24) may correspond to a radiative fluid (Weinberg [2]). Near the big bang, $0 \le n \le \frac{1}{2}$ is a more appropriate assumption (Belinskii and Khalatnikov [47]) to obtain realistic models.

For simplicity sake and for realistic models of physical importance, we consider the following two cases ($n = 0, 1$).

3.1.1. **Model I: Solution for $n = 0$**

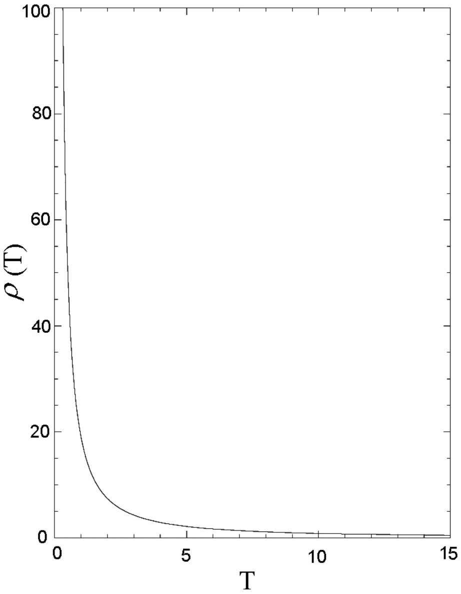
When $n = 0$, Eq. (24) reduces to $\xi = \xi_0 =$ constant. Hence in this case Eq. (21), with the use of (22) and (23), leads to

$$8\pi(1+\gamma)\rho = \frac{8\pi L(3\xi_0 - 2\eta_0)(8\pi\eta_0 t + a)}{3(Lt+M)} + \frac{(L^2-b^2)(8\pi\eta_0 t + a)^2}{2(Lt+M)^2} - 129\pi^2\xi_0\eta_0 \,. \tag{25}$$

Eliminating $\rho(t)$ between (22) and (25), we obtain

$$(1+\gamma)\Lambda = -\frac{8\pi L(3\xi_0 + 4\eta_0 + 6\eta_0\gamma)(8\pi\eta_0 t + a)}{3(Lt+M)} + \frac{(L^2-b^2)(8\pi\eta_0 t + a)^2(1-\gamma)}{4(Lt+M)^2} - 129\pi^2\eta_0(\xi_0 + \eta_0 + \eta_0\gamma) \,. \tag{26}$$

From Eq. (25), we note that for $t > (a - M)/(L - 8\pi\eta_0)$, $\rho(t)$ is a decreasing function of time and $\rho > 0$ for all times. Figure 1 shows this behaviour of energy density.



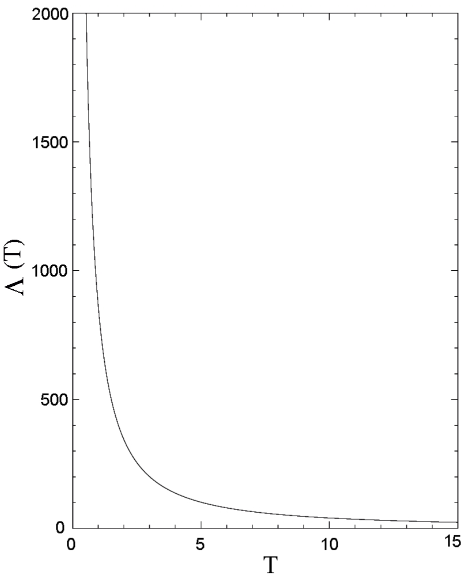


Fig. 1 (left). The plot of energy density $\rho(t)$ vs. time.
Fig. 2. The plot of cosmological term $\Lambda(t)$ vs. time.

The behaviour of the universe in this model will be determined by the cosmological term Λ, this term has the same effect as a uniform mass density $\rho_{\text{eff}} = -\Lambda/4\pi G$ which is constant in space and time. A positive value of Λ corresponds to a negative effective mass density (repulsion). Hence, we expect that in the universe with a positive value of Λ the expansion will tend to accelerate, whereas in the universe with negative value of Λ the expansion will slow down, stop and reverse. In a universe with both matter and vacuum energy, there is a competition between the tendency of Λ to cause acceleration and the tendency of matter to cause deceleration with the ultimate fate of the universe depending on the precise amounts of each component. This continues to be true in the presence of spatial curvature, and with a nonzero cosmological constant it is no longer true that the negatively curved ("open") universes expand indefinitely while positively curved ("closed") universes will necessarily recollapse - each of the four combinations of negative or positive curvature and eternal expansion or eventual recollapse become possible for appropriate values of the parameters. There may even be a delicate balance, in which the competition between matter and vacuum energy is needed drawn and the universe is static (non expanding). The search for such a solution was Einstein's original motivation for introducing the cosmological constant.

From Eq. (26), we see that for $t > (a - M)/(L - 8\pi\eta_0)$, the cosmological term Λ is a decreasing function of time and it approaches a small positive value as time increases. From Fig. 2 we note this behaviour of Λ. Recent cosmological observations (Garnavich et al. [21], Perlmutter et al. [18], Riess et al. [19], Schmidt et al. [24]) suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. These observations on magnitude and red-shift of type Ia supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological Λ-term. Thus, our model is consistent with the results of recent observations.

3.1.2. **Model II: Solution for $n = 1$**

When $n = 1$, Eq. (24) reduces to $\xi = \xi_0 \rho$. Hence in this case Eq. (21), with the use of (22) and (23), leads to

$$8\pi\left[1 + \gamma + \xi_o\left\{\frac{8\pi\eta_0(2Lt + 3M) - aL}{(Lt + M)}\right\}\right]\rho =$$

$$\frac{(L^2 - b^2)(8\pi\eta_0 t + a)^2}{2(Lt + M)^2} - \frac{16\pi\eta_0 L(8\pi\eta_0 t + a)}{3(Lt + M)}\,. \tag{27}$$

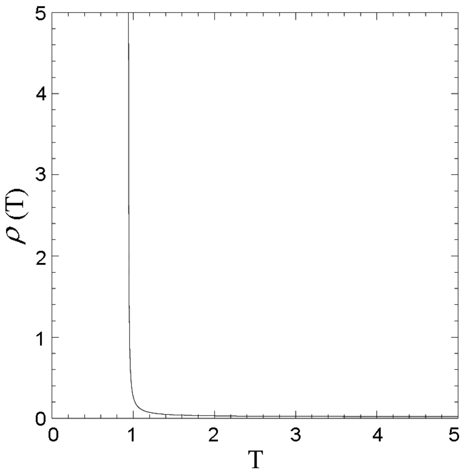
Eliminating $\rho(t)$ between (22) and (27), we obtain

$$\left[1 + \gamma + \xi_o\left\{\frac{8\pi\eta_0(2Lt + 3M) - aL}{(Lt + M)}\right\}\right]\Lambda = \frac{(L^2 - b^2)(8\pi\eta_0 t + a)^2}{2(Lt + M)^2}$$

$$-\frac{16\pi\eta_0 L(8\pi\eta_0 t+a)}{3(Lt+M)} - \left[1+\gamma+\xi_o\left\{\frac{8\pi\eta_0(2Lt+3M)-aL}{(Lt+M)}\right\}\right]\times$$

$$\left[192\pi^2\eta_0^2 + \frac{(L^2-b^2)(8\pi\eta_0 t+a)^2}{4(Lt+M)^2} - \frac{16\pi\eta_0 L(8\pi\eta_0 t+a)}{(Lt+M)}\right]. \tag{28}$$

From Eq. (27), we observe that for $t > (a-M)/(L-8\pi\eta_0)$, $\rho(t)$ is a decreasing function of time and $\rho > 0$ for all times. Figure 3 shows this behaviour of energy density. From Eq. (28), we note that $t > (a-M)/(L-8\pi\eta_0)$, the cosmological term Λ is a decreasing function of time and it approaches a small positive value with increase in time. From Fig. 4 we note the same character of Λ. This is consistent with recent observations (Garnavich et al. [21], Perlmutter et al. [18], Riess et al. [19], Schmidt et al. [24]).



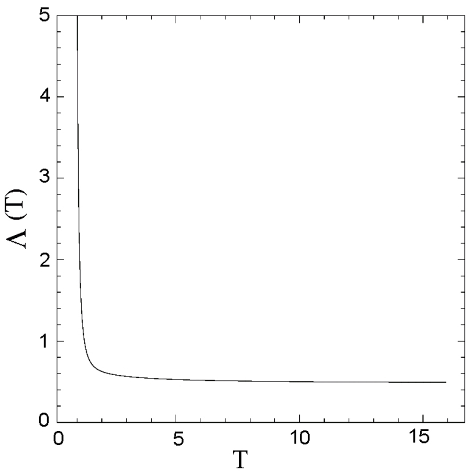


Fig. 3 (left). The plot of energy density $\rho(t)$ vs. time.

Fig. 4. The plot of cosmological term $\Lambda(t)$ vs. time.

3.1.3. Some geometric properties of first model

We shall now give the expressions for kinematic quantities and components of conformal curvature tensor. With regard to the kinematical properties of the velocity vector v^i in the metric (20), a straightforward calculation leads to the expressions for expansion (θ), the deceleration parameter (q), the proper volume (V^3) and shear (σ_{ij}) of the fluid:

$$\theta = \frac{(8\pi\eta_0 t+a)L}{(Lt+M)} - 24\pi\eta_0, \tag{29}$$

$$q = -1 - \frac{\dfrac{256\pi^2\eta_0^2}{(8\pi\eta_0 t + a)^2} - \dfrac{L^2}{(Lt+M)^2}}{\dfrac{L^2}{(Lt+M)^2} + \dfrac{(32\pi\eta_0)^2}{(8\pi\eta_0 t + a)^2} - \dfrac{64\pi\eta_0 L}{(8\pi\eta_0 t + a)(Lt+M)}}\,, \tag{30}$$

$$V^3 = \sqrt{-g} = \frac{\left[K^2(Lt+M) - b^2/(4L)\right]}{(8\pi\eta_0 + a)K^2(8\pi\eta_0 + a)}\,, \tag{31}$$

$$\sigma_{11} = -\frac{L(Lt+M)^{-1}}{3(8\pi\eta_0 t + a)}\,, \tag{32}$$

$$\sigma_{22} = \frac{K^2(L+3b)(Lt+M)^{\frac{b}{L}}}{6(8\pi\eta_0 t + a)}\,, \tag{33}$$

$$\sigma_{33} = \frac{(L-3b)(Lt+M)^{-\frac{b}{L}}}{6K^2(8\pi\eta_0 t + a)}\,, \tag{34}$$

and other components of the shear tensor (σ_{ij}) being zero. Hence

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \left(\frac{(L^2+3b^2)}{12}\right)\left(\frac{8\pi\eta_0 t + a}{Lt+M}\right)^2. \tag{35}$$

From Eqs. (29) and (35) we obtain

$$\frac{\sigma}{\theta} = \frac{\sqrt{(L^2+3b^2)/12}\;(8\pi\eta_0 t + a)}{(8\pi\eta_0 t + a)L - 24\pi\eta_0(Lt+M)}\,. \tag{36}$$

The non-vanishing components of the conformal curvature tensor are

$$C^{12}{}_{12} = C^{13}{}_{13} = -\frac{1}{2}C^{23}{}_{23} = \left(\frac{(L^2-b^2)}{12}\right)\left(\frac{8\pi\eta_0 t + a}{Lt+M}\right)^2. \tag{37}$$

For large t, we find

$$C^{23}{}_{23} = -\frac{32}{3}\pi^2\eta_0^2\left(1 - \frac{b^2}{L^2}\right) \tag{38}$$

and

$$\sigma^2 = \frac{16}{3}\pi^2\eta_0^2\left(1 + \frac{3b^2}{L^2}\right). \tag{39}$$

Here we find

$$C^{12}{}_{12} + C^{13}{}_{13} + C^{23}{}_{23} = 0\,. \tag{40}$$

The rotation ω is identically zero.

The rate of expansion H_i in the directions of x, y and z are given by

$$H_x = \frac{\dot{A}}{A} = -\frac{8\pi\eta_0}{(8\pi\eta_0 t + a)}\,, \tag{41}$$

$$H_y = \frac{\dot{B}}{B} = \left(\frac{1}{2} + \frac{b}{2L}\right)\frac{L}{(Lt+M)} - \frac{8\pi\eta_0}{(8\pi\eta_0 t + a)}\,, \tag{42}$$

$$H_z = \frac{\dot{C}}{C} = \left(\frac{1}{2} - \frac{b}{2L}\right)\frac{L}{(Lt+M)} - \frac{8\pi\eta_0}{(8\pi\eta_0 t + a)}\,. \tag{43}$$

Since

$$\int_{t_0}^{t} \frac{\mathrm{d}t}{V(t)} = \int_{t_0}^{t} \frac{(8\pi\eta t + a)^{\frac{4}{3}}}{(Lt+M)^{\frac{1}{3}}}\mathrm{d}t\,, \tag{44}$$

this is convergent integral, hence the particle horizon exists.

The models represent shearing, non-rotating and Petrov type D universe in general, in which the flow is geodetic. It is also observed that the viscosity prevents the free gravitational field as well as the shear from withering away. It is also obvious from (29) that the effect of viscosity is to retard expansion of the model. Since $\lim_{t\to\infty} \sigma/\theta \neq 0$, the models do not approach isotropy for large values of t. It is observed from Eq. (30) which implies an accelerating model of the universe. Recent observations of type Ia supernovae [39, 40] reveal that the present universe is in accelerating phase and deceleration parameter lies somewhere in the range $-1 < q \leq 0$. It follows that our models of the universe are consistent with the recent observations. For

$$256\pi^2\eta^2(Lt+M)^2 = L^2(8\pi\eta t + a)^2\,, \tag{45}$$

the deceleration parameter q approaches the value (-1) as in the case of de-Sitter universe.

3.2. The second model

The condition

$$C^{12}{}_{12} = C^{23}{}_{23}, \tag{46}$$

leads to

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{A}}{A}\right) = \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right). \tag{47}$$

Equations (10), (11) and (47) lead to

$$\frac{\dot{B}}{B} = -8\pi\eta_0 A \,. \tag{48}$$

From Eqs. (11) and (48) we obtain

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} = 2\frac{\dot{B}}{B}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right), \tag{49}$$

which on integration leads

$$C = B(k_1 - kt)\,, \tag{50}$$

where k and k_1 are constants of integration. From Eqs. (47) and (50) we get

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\left(\frac{k}{k_1 - kt}\right), \tag{51}$$

which on integration gives

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_2}{k_1 - kt}\,, \tag{52}$$

where k_2 is an integrating constant. From Eqs. (48) and (52) we obtain

$$A = \left[\frac{8\pi\eta_0(k_1 - kt)}{k_2 - k} + k_3(k_1 - kt)^{\frac{k_2}{k}}\right]^{-1}, \tag{53}$$

k_3 being a constant of integration. From Eqs. (48) and (53) we obtain

$$B = k_4\left[\frac{(k_2 - k)k_3(k_1 - kt)^{\frac{k_2}{k}-1}}{8\pi\eta_0 + (k_2 - k)k_3(k_1 - kt)^{\frac{k_2}{n}-1}}\right], \tag{54}$$

where k_4 is a constant of integration. Also, from Eqs. (50) and (54) we obtain

$$C = k_4\left[\frac{(k_2 - k)k_3(k_1 - kt)^{\frac{k_2}{k}}}{8\pi\eta_0 + (k_2 - k)k_3(k_1 - kt)^{\frac{k_2}{n}-1}}\right]. \tag{55}$$

By a suitable transformation of coordinates, the metric of this model can be put into the form

$$\mathrm{d}s^2 = \left(\frac{8\pi\eta_0}{\alpha - 1}T + \beta T^{\alpha}\right)^{-2}\left[\mathrm{d}X^2 - \mathrm{d}T^2 + T^{2\alpha}\mathrm{d}Y^2 + T^{2(\alpha+1)}\mathrm{d}Z^2\right], \tag{56}$$

where α and β are arbitrary constants.

The distribution of pressure (p) and density (ρ) in the model (56) is given by

$$8\pi p = 64\pi^2\eta_0^2\left(\frac{2-\alpha}{\alpha-1}\right) + 16\pi\eta_0\alpha\beta T^{\alpha-1} + \alpha(\alpha-1)\beta^2T^{2(\alpha-1)}$$

$$-\left(\frac{8\pi\eta_0}{\alpha-1} + \alpha\beta T^{\alpha-1}\right)^2 - \frac{16}{3}\pi\eta_0(\alpha+2)\left(\frac{8\pi\eta_0}{\alpha-1} + \beta T^{\alpha-1}\right)$$

$$+8\pi\xi[(\alpha-1)\beta T^{\alpha-1} - 16\pi\eta_0] - \Lambda\,, \tag{57}$$

$$8\pi\rho = 64\pi^2\eta_0^2\left[1 - \frac{\alpha}{(\alpha-1)^2}\right] + 16\pi\eta_0\beta\left(1 - \frac{\alpha^2}{\alpha-1}\right)T^{\alpha-1}$$

$$-\alpha\beta^2T^{2(\alpha-1)} + \Lambda\,. \tag{58}$$

For simplicity sake and for realistic models of physical importance, we again consider the following two cases ($n = 0, 1$):

3.2.1. **Model I: Solution for $n = 0$**

When $n = 0$, Eq. (24) reduces to $\xi = \xi_0 =$ constant. Hence, in this case Eq. (57), with the use of (58) and (23), leads to

$$8\pi(1+\gamma)\rho = -\frac{128\pi^2\eta_0^2(\alpha^2+\alpha+1)}{3(\alpha-1)^2} - \frac{16\pi\beta\eta_0(\alpha+1)}{(\alpha-1)}T^{\alpha-1}$$

$$-2\alpha\beta^2T^{2(\alpha-1)} - \frac{16}{3}\pi\beta\eta_0(\alpha+2)T^{\alpha-1} + 8\pi\xi_0[(\alpha-1)\beta T^{\alpha-1} - 16\pi\eta_0]\,. \tag{59}$$

Eliminating $\rho(t)$ between (58) and (59), we obtain

$$(1+\gamma)\Lambda = -\frac{128\pi^2\eta_0^2(\alpha^2+\alpha+1)}{3(\alpha-1)^2} - \frac{16\pi\beta\eta_0(\alpha+1)}{(\alpha-1)}T^{\alpha-1} - 2\alpha\beta^2T^{2(\alpha-1)}$$

$$+8\pi\xi_0[(\alpha-1)\beta T^{\alpha-1} + \alpha\beta^2(1+\gamma)T^{2(\alpha-1)} - 16\pi\eta_0] - 64\pi^2\eta_0^2(1+\gamma)\left\{1 - \frac{\alpha}{(\alpha-1)^2}\right\}$$

$$-\frac{16}{3}\pi\beta\eta_0(\alpha+2)T^{\alpha-1} - 16\pi\eta_0\beta(1+\gamma)\left(1 - \frac{\alpha^2}{\alpha-1}\right)T^{\alpha-1}. \tag{60}$$

From Eq. (59), we see that for $\alpha < 0$ and $\beta < 0$, $\rho(t)$ is a decreasing function of time and $\rho > 0$ for all times. Figure 5 shows this behaviour of energy density. From

Eq. (60), we note that the cosmological term Λ is a decreasing function of time and it approaches a small positive value at late time for $\alpha < 0$, $\beta < 0$, $\rho(t)$. From Fig. 6, we note the same character of Λ. This is consistent with recent observations (Garnavich et al. [21], Perlmutter et al. [18], Riess et al. [19], Schmidt et al. [24]).

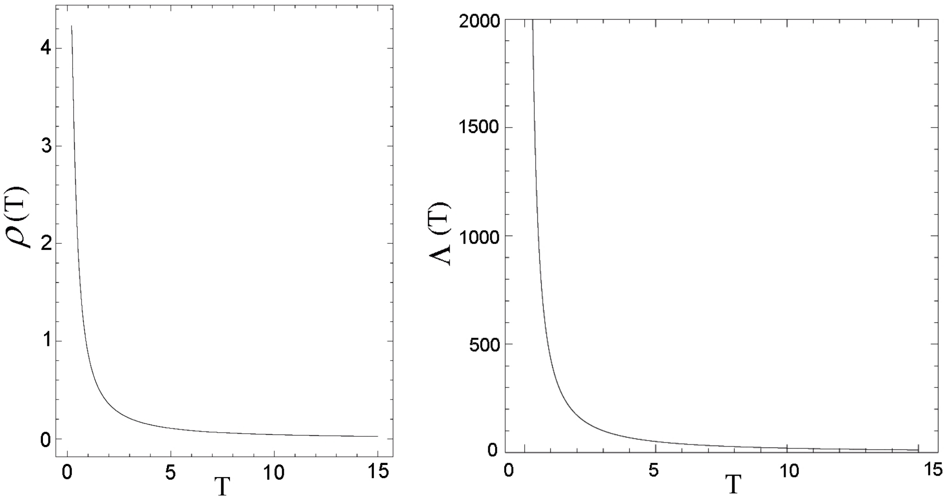


Fig. 5 (left). The plot of energy density $\rho(t)$ vs. time.
Fig. 6. The plot of cosmological term $\Lambda(t)$ vs. time.

3.2.2. Model II: Solution for $n = 1$

When $n = 1$, Eq. (24) reduces to $\xi = \xi_0 \rho$. Hence in this case Eq. (57), with the use of (58) and (23), leads to

$$8\pi\Big[1+\gamma-\xi_0\left\{(\alpha-1)\beta T^{\alpha-1}-16\pi\eta_0\right\}\Big]\rho = -\frac{128\pi^2\eta_0^2(\alpha^2+\alpha+1)}{3(\alpha-1)^2}$$

$$-\frac{16\pi\beta\eta_0(\alpha+1)}{(\alpha-1)}T^{\alpha-1}-2\alpha\beta^2T^{2(\alpha-1)}-\frac{16}{3}\beta\eta_0(\alpha+2)T^{\alpha-1}. \tag{61}$$

Eliminating $\rho(t)$ between (58) and (61), we obtain

$$\Big[1+\gamma-\xi_0\left\{(\alpha-1)\beta T^{\alpha-1}-16\pi\eta_0\right\}\Big]\Lambda = -\frac{128\pi^2\eta_0^2(\alpha^2+\alpha+1)}{3(\alpha-1)^2}$$

$$-\frac{16\pi\beta\eta_0(\alpha+1)}{(\alpha-1)}T^{\alpha-1}-2\alpha\beta^2T^{2(\alpha-1)}-\frac{16}{3}\beta\eta_0(\alpha+2)T^{\alpha-1}-$$

$$\Big[1+\gamma-\xi_0\left\{(\alpha-1)\beta T^{\alpha-1}-16\pi\eta_0\right\}\Big]\times\Bigg[64\pi^2\eta_0^2\left\{1-\frac{\alpha}{(\alpha-1)^2}\right\}$$

$$+16\pi\eta_0\beta\left(1-\frac{\alpha^2}{\alpha-1}\right)T^{\alpha-1}-\alpha\beta^2T^{2(\alpha-1)}\Bigg]. \tag{62}$$

From Eq. (61), we see that for $\alpha < 0$ and $\beta < 0$, $\rho(t)$ is a decreasing function of time and $\rho > 0$ for all times. From Fig. 7 we observe the same nature of ρ. From Eq. (62), we note that the cosmological term Λ is a decreasing function of time and it approaches a small positive value at late time for $\alpha < 0$, $\beta < 0$. From Fig. 8 we note the same behaviour of Λ. This is consistent with recent observations (Garnavich et al. [21], Perlmutter et al. [18], Riess et al. [19], Schmidt et al. [24]).

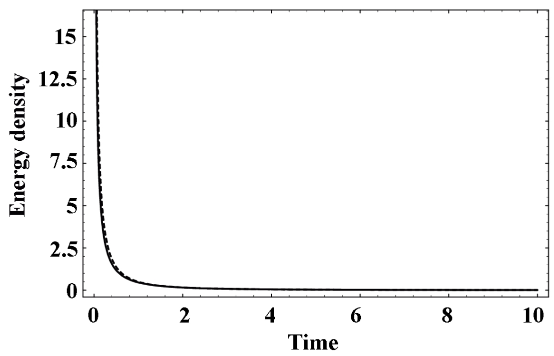


Fig. 7. The plot of cosmological term $\rho(t)$ vs. time.

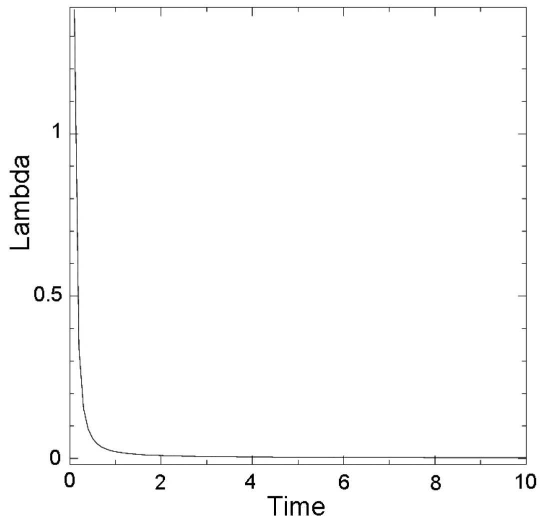


Fig. 8. The plot of cosmological term $\Lambda(t)$ vs. time.

3.2.3. Some geometric properties of the second model

The expressions for the expansion θ, Hubble parameter H, the magnitude of shear σ^2, deceleration parameter q and proper volume V^3 for the model (56) are given by

$$\theta = 3H = \beta(\alpha-1)T^{\alpha-1} - 16\pi\eta_0 \,, \tag{63}$$

$$\sigma^2 = \frac{1}{3}(\alpha^2+\alpha+1)\left[\frac{8\pi\eta_0}{\alpha-1} + \beta T^{\alpha-1}\right]^2, \tag{64}$$

$$q = -1 - \frac{1}{\left[\frac{(2\alpha+1)}{3T} - \frac{2\beta(\alpha-1)T^{\alpha-1}}{3\left\{8\pi\eta_0 + \beta(\alpha-1)T^{\alpha}\right\}}\right]^2} \times$$

$$\left[-\frac{(2\beta+1)}{3T^2} + \frac{2\beta(\alpha-1)^2(8\pi\eta_0 T^{\alpha-2} + \beta\alpha T^{\alpha-1})}{3\left\{8\pi\eta_0 T^{\alpha-1} + \beta(\alpha-1)T^{\alpha}\right\}^2}\right], \tag{65}$$

$$V^3 = \sqrt{-g} = \left[\frac{8\pi\eta_0}{(\alpha-1)}T + \beta T^{\alpha}\right]^{-2} T^{2\alpha+1}. \tag{66}$$

The non-vanishing components of the conformal curvature tensor are

$$C^{12}{}_{12} = C^{23}{}_{23} = -\frac{1}{2}C^{13}{}_{13} = -\frac{1}{3}\alpha\left[\frac{8\pi\eta_0}{\alpha-1} + \beta T^{\alpha-1}\right]. \tag{67}$$

Here we also find

$$C^{12}{}_{12} + C^{13}{}_{13} + C^{23}{}_{23} = 0\,. \tag{68}$$

The rotation ω is identically zero.

The rate of expansion H_i in the directions of x, y and z are given by

$$H_x = \frac{\dot{A}}{A} = -\frac{\left(8\pi\eta_0/(\alpha-1) + \alpha\beta T^{\alpha-1}\right)}{\left(8\pi\eta_0/(\alpha-1)\; T + \beta T^{\alpha}\right)}\,, \tag{69}$$

$$H_y = \frac{\dot{B}}{B} = \frac{\alpha}{T} - \frac{\left(8\pi\eta_0/(\alpha-1) + \alpha\beta T^{\alpha-1}\right)}{\left(8\pi\eta_0/(\alpha-1)\; T + \beta T^{\alpha}\right)}\,, \tag{70}$$

$$H_z = \frac{\dot{C}}{C} = \frac{(\alpha+1)}{T} - \frac{\left(8\pi\eta_0/(\alpha-1) + \alpha\beta T^{\alpha-1}\right)}{\left(8\pi\eta_0/(\alpha-1)\; T + \beta T^{\alpha}\right)}\,. \tag{71}$$

The models represent shearing, non-rotating and Petrov type D universe in general, in which the flow is geodetic. For this model too, it is observed that the effect of viscosity prevents the shear and the free gravitational field from withering away for large value of T. It also retards expansion of the model. Since $\lim_{T\to\infty} \sigma/\theta \neq 0$, the models do not approach isotropy for large values of T. It is observed from Eq. (65) which implies an accelerating model of the universe. It follows that our models of the universe are consistent with recent observations [18, 19, 21, 24]. For the critical time T_c given by

$$\frac{(2\beta+1)}{3T_c^2} = \frac{2\beta(\alpha-1)^2(8\pi\eta_0 T_c^{\alpha-2} + \beta\alpha T_c^{\alpha-1})}{3\left\{8\pi\eta_0 T_c^{\alpha-1} + \beta(\alpha-1)T_c^{\alpha}\right\}^2}\,, \tag{72}$$

the deceleration parameter q approaches the value (-1) as in the case of de-Sitter universe.

We also find

$$\int_{t_0}^{t} \frac{\mathrm{d}t}{V(t)} = \int_{t_0}^{t} \frac{[8\pi\eta/(\alpha-1)\ T + \ell T^{\alpha}]^{\frac{2}{3}}}{T^{(2\alpha+1)/3}} \mathrm{d}t\,, \tag{73}$$

which is the convergent integral and hence particle horizon exists.

The metric (49) is conformal to the metric

$$\mathrm{d}s^2 = \mathrm{d}X^2 - \mathrm{d}T^2 + T^{2\alpha}\mathrm{d}Y^2 + T^{2(\alpha+1)}\mathrm{d}Z^2. \tag{74}$$

The universe (74) represents a viscous fluid cosmological model in which kinematic viscosity η_{00} is $-\alpha/(8\pi T)$ and the pressure p_0 and the density ρ_0 are given by

$$8\pi p_0 = 8\pi\xi\left(\frac{2\alpha+1}{T}\right) - \left[\frac{\alpha(5\alpha+1)}{3T^2}\right] - \Lambda\,, \tag{75}$$

$$8\pi\rho_0 = \frac{\alpha(\alpha+1)}{T^2} + \Lambda\,. \tag{76}$$

It is also remarkable that the space-time (74) becomes flat when α is zero. The corresponding model

$$\mathrm{d}s^2 = (\beta - 8\pi\eta_0 T)^{-2}(\mathrm{d}X^2 - \mathrm{d}T^2 + \mathrm{d}Y^2 + T^2\mathrm{d}Z^2) \tag{77}$$

represents a conformally flat viscous fluid cosmological model.

4. *Discussion and conclusion*

We have presented a new class of plane-symmetric cosmological models of the universe with a viscous fluid as the source of matter which incorporates a vacuum energy-density term decaying with time. Generally, the models represent shearing, non-rotating and Petrov type D universe in which the flow vector is geodetic. In all these models, we observe that they do not approach isotropy for large values of time.

Recent observations of distant supernovae imply, in defiance of expectation, that the universe growth is accelerating, contrary to what has always been assumed that the expansion is slowing down due to gravity. If in the light of these observations Λ is a non-zero, we will be faced with the additional task of inventing a theory which sets the vacuum energy density to be a very small value without setting it precisely at zero. In this case we may distinguish between a "true" vacuum, which would be the state of lowest possible energy that is non-zero and a "false" vacuum being metastably different from the actual state of lowest energy (which might well have $\Lambda = 0$). Such a state could eventually decay into the true vacuum, whereas its life-time could be much larger than the current age of the universe. A final possibility is that the vacuum energy density is changing with time - a dynamical cosmological "constant". In the present theoretical study, the cosmological constants in all these models given in Subsections 3.1 and 3.2, are decreasing functions of time and approach a small value as time increases (i.e. the present epoch). The values of cosmological "constant" for these models are found to be small and positive which is supported by the results from recent supernovae Ia observations recently obtained by the High-z Supernova Team and Supernova Cosmological Project (Garnavich et al. [21] , Perlmutter et al. [18], Riess et al. [19], Schmidt et al. [24]). Our derived models confirm these recent experimental results by showing that the universe now is definitely in a stage of accelerating expansion. Thus, with our approach, we obtain a physically relevant decay law for the cosmological constant unlike other investigators where *ad hoc* laws were used to arrive at a mathematical expressions for the decaying vacuum energy. Thus, our models are more general than those studied earlier. Our solutions generalize the solutions obtained by Roy and Prakash [40].

The effect of bulk viscosity is to produce a change in perfect fluid and therefore exhibits essential influence on the character of the solution. We also observe here that Murphy's [46] conclusion about the absence of a big bang type singularity in the infinite past in models with bulk viscous fluid in general, is not true. The results obtained by Myung and Cho [48] also show that, it is not generally valid since for some cases big bang singularity occurs in finite past. For both models, it is observed that the effect of viscosity prevents the shear and the free gravitational field from withering away.

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RAVNINSKI SIMETRIČAN, VISKOZAN I FLUIDAN SVEMIR S OPADAJUĆOM GUSTOĆOM ENERGIJE Λ

Istražujemo ravninski simetrične, viskozne i fluidne modele svemira s promjenljivom kozmološkom konstantom. Pretpostavlja se da je koeficijent viskoznosti volumno viskoznog fluida dan s gustoćom mase na neku potenciju, a uzima se konstantan koeficijent posmične viskoznosti. Da bi se dobila određena rješenja Einsteinovih jednadžbi polja, pretpostavlja se da je slobodno gravitacijsko polje tipa D što je sljedeći red u Petrovoj klasifikaciji. Dobiva se s vremenom opadajuća ali pozitivna kozmološka konstanta Λ što je u skladu s nedavnim opažanjima supernova Ia. Raspravlja se fizikalni i geometrijski značaj modela.