Printed ISSN 1330–0016
Online ISSN 1333–9133
CD ISSN 1333–8390
CODEN FIZBE7

PRODUCTION OF f_0-MESON IN THE PROCESS $e^+e^- \to \varphi f_0$

YU. M. BYSTRITSKIY[a], M. K. VOLKOV[a], E. A. KURAEV[a], E. BARTOŠ[b] and M. SEČANSKÝ[b]

[a]*Joint Institute for Nuclear Research, Dubna, Russia*

[b]*Institute of Physics, Slovak Academy of Sciences, Bratislava*

E-mail addresses: bystr@theor.jinr.ru, volkov@theor.jinr.ru, kuraev@theor.jinr.ru, 3rd@jinr.ru

Received 23 October 2007; Accepted 20 February 2008
Online 20 June 2008

Enigmatic nature of $f_0(980)$ meson can be tested in C–τ factory with total energy in e^+e^- center of mass $3 < \sqrt{s} < 4$ GeV. This facility has the advantage compared with φ–factory with $\sqrt{s} = 1.02$ GeV where $f_0(980)$ properties can be investigated in the process $e^+e^- \to \gamma f_0(980)$. The reason of that is strong phase volume suppression $\sim \omega^3$ of the cross section in the experiments at φ–factories. Our approach is based on the analysis of the lowest order loop Feynman diagrams with the strange constituent quark or charged kaon as virtual loop particles.

The relative values of the contributions of quark loop and kaon loop are discussed. The differential cross section as well as the estimation of the total one are illustrated numerically.

PACS numbers: 13.60 Le UDC 539.126

Keywords: meson, NJL model

1. Introduction

Nontrivial nature of isovector meson $a_0(980)$ and isoscalar meson $f_0(980)$ now is in active investigation (see Refs [1, 2] and references therein).

Some properties of these resonances can not be explained in the frames of the naive quark-antiquark model [3, 4]. The equality of masses is similar to the ones of ρ and ω resonances and lead to the assumption about a similar structure, but the relation of $f_0(980)$ with $\bar{K}K$ channel points out to a probable presence of strange quarks $\bar{s}s$ in f_0.

Present models, without and with strange quarks as well as four-quark model and the one with $\bar{K}K$-molecule types were intensively discussed in the literature.

In Ref. [5] it was shown that critical experiment for the structure of a_0, f_0 can be the radiative decays of φ-meson $\varphi \to f_0(980)\gamma$ $(a_0(980)\gamma) \to \pi^0\pi^0\gamma$ $(\eta\pi^0\gamma)$.

The predictions for the width of different models are quite different. Experimental values of the relevant branching ratios are approximately $Br(\varphi \to \eta\pi^0\gamma) = 8 \cdot 10^{-5}$ and $Br(\varphi \to \pi^0\pi^0\gamma) = 1.2 \cdot 10^{-4}$ which were first obtained at the Novosibirsk φ-factory [6]. KLOE Collaboration [7] obtained $Br(\varphi \to \pi^0\pi^0\gamma) = 10.9 \cdot 10^{-5}$ and $Br(\varphi \to \eta\pi^0\gamma) = 8 \cdot 10^{-5}$.

Much attention to the structure of $f_0(980)$ meson was paid since 1990 [5] when the arguments in favour of its molecular nature $f_0(980) = [(u\bar{u} + d\bar{u})/\sqrt{2}]s\bar{s}$ were given. The experimental support of this was found in the φ–factory [9], but unfortunately with rather small statistics. The reason why this molecular model was elaborated is the fact that the contribution of the triangle–type amplitude describing $\gamma^* \to \varphi \to \gamma f_0(980)$ subprocess with the strange quark inside a loop was negligible in comparisone with the triangle amplitude with the charged kaon in the loop. Now much attention is paid to the radiative decays of light scalar mesons which are considered within the ChPT framework (see for example Ref. [8]).

Due to the low-energy theorem, the matrix element of the subprocess $\gamma^* \to \gamma f_0(980)$ must vanish for the real photon energy $\omega \to 0$. So differential cross section of the process $e^+e^- \to \gamma f_0(980)$ must be proportional to ω^3. Such a kind of the behaviour is measured by the experiments at the φ–factory. Unfortunately, the phase volume for $\sqrt{s} \equiv m_\varphi = 1020$ MeV is small. This leads to a small statistics. For instance, at the VEPP–2M facility it did not exceed 20 events. So in this paper we want to attract the attention to the possibility of investigation of the nature of $f_0(980)$ meson as well as similar ones in crossed channels (when the real photon becomes highly virtual) obtained in e^+e^- annihilation.

2. *Subprocess* $\gamma^* \to \varphi f_0(980)$

The amplitude of the process $\gamma^* \to \varphi f_0(980)$ consists of two terms

$$M\left(\gamma^*(q_1,\mu) \to \varphi(q_2,\nu) f_0(q_3)\right) = \\ = \sum_{i=q,K} \frac{C_{(i)}}{2^4\pi^2} \varepsilon_\mu(q_1)\, \varepsilon_\nu(q_2) \Big(A_{(i)}\, R^{\mu\nu}_{(1)}(q_1,q_2) + B_{(i)}\, R^{\mu\nu}_{(2)}(q_1,q_2) \Big), \quad (1)$$

where i denotes the type of a contribution ($i = q$ corresponds to the quark–loop contribution, and $i = K$ to the kaon–loop contribution), $R^{\mu\nu}_{(j)}(q_1,q_2)$ are gauge invariant structures

$$\begin{aligned} R^{\mu\nu}_{(1)}(q_1,q_2) &= g^{\mu\nu}\,(q_1q_2) - q_1^\nu q_2^\mu, \\ R^{\mu\nu}_{(2)}(q_1,q_2) &= \left(q_1 - q_2\frac{q_1^2}{(q_1q_2)}\right)^\mu \left(q_2 - q_1\frac{q_2^2}{(q_1q_2)}\right)^\nu, \\ R^{(i)}_{\mu\nu}(q_1,q_2)\, q_1^\mu &= R^{(i)}_{\mu\nu}(q_1,q_2)\, q_2^\nu = 0, \qquad i = 1,2 \end{aligned} \quad (2)$$

$A_{(i)}$, $B_{(i)}$ are the functions of momentum squares only (q_1^2, q_2^2, q_3^2) and $C_{(i)}$ are the multiplication of vertex constants of the loop.

2.1. Quark loop contribution

We define the quark–meson vertexes in the following way

$$\begin{aligned}
\Gamma^{\mu}_{\gamma\to q\bar{q}} &= -\mathrm{i}\, e\, Q\gamma^{\mu},\\
\Gamma^{\mu}_{\varphi\to q\bar{q}} &= -\mathrm{i}\,\frac{g_\rho}{2}\lambda_s\gamma^{\mu},\\
\Gamma_{f_0\to q\bar{q}} &= (-\mathrm{i})\left(g_{\sigma_s}\lambda_s\cos(\bar{\varphi}) + g_{\sigma_u}\lambda_u\sin(\bar{\varphi})\right),
\end{aligned}$$

where $e = |e|$ is the proton charge, $Q = \mathrm{diag}\{2/3, -1/3, -1/3\}$ is the quark charge matrix, $\lambda_s = (-\lambda_0 + \sqrt{2}\lambda_8)/\sqrt{3}$, $\lambda_u = (\sqrt{2}\lambda_0 + \lambda_8)/\sqrt{3}$, where λ_i are the Gell–Mann matrixes, $\bar{\varphi} = 11.26^\circ$ is the mixing angle in the scalar sector and $g_\rho = 5.938$, $g_{\sigma_s} = 2.988$, $g_{\sigma_u} = 2.424$.

Then the amplitude of quark–loop contribution has the form

$$M^{(q)}(\gamma^*(q_1,\mu)\to\varphi(q_2,\nu)f_0(q_3)) = \frac{3\mathrm{i}\,e}{(2\pi)^4}\frac{g_\rho}{2}\left(-\frac{2}{3}\right) g_{\sigma_s}\cos(\bar{\varphi})\varepsilon_\mu(q_1)\varepsilon_\nu(q_2) \qquad (3)$$

$$\times\int \mathrm{d}k\,\frac{\mathrm{Tr}\left[\gamma^\mu(\hat{k}-\hat{q}_2+m_s)(\hat{k}+m_s)\gamma^\nu(\hat{k}+\hat{q}_1+m_s)\right]}{((k-q_2)^2-m_s^2)\,(k^2-m_s^2)\,((k+q_1)^2-m_s^2)},$$

where we have calculated the trace over Gell–Mann matrixes and $m_s = 0.407$ GeV is the mass of the strange quark. After calculation of the trace and loop momenta integration, with the use of the Feynman parametrization method, the amplitude (4) can be rewritten in the form (1), where

$$A_{(q)} = \int_0^1 \mathrm{d}x\int_0^1 \frac{y\mathrm{d}y}{D_q}\left(-\alpha_q + \frac{q_1^2q_2^2}{(q_1q_2)^2}\beta_q\right),\ B_{(q)} = \int_0^1 \mathrm{d}x\int_0^1 \frac{y\mathrm{d}y}{D_q}\beta_q,\ C_{(q)} = e\,g_\rho g_{\sigma_s}\cos(\bar{\varphi}) \qquad (4)$$

where

$$\begin{aligned}
&\alpha_q = 4m_s\left(1-4xy\bar{y}\right), \qquad \beta_q = 4m_s(1-2y(x-2xy+1)),\\
&D_q = m_s^2 + q_x^2y^2 - yh_x, \quad q_x^2 = q_3^2x + q_2^2\bar{x} - q_1^2x\bar{x}, \quad h_x = q_2^2\bar{x} + q_3^2x, \qquad (5)
\end{aligned}$$

with $\bar{x} = 1-x$, $\bar{y} = 1-y$.

2.2. Kaon loop contribution

For the calculation of the kaon loop, the following vertices are needed:

$$\begin{aligned}
\Gamma^{\mu}_{\gamma\to K^+(p_+)K^-(p_-)} &= -\mathrm{i}\,e(p_+ + p_-)^\mu,\\
\Gamma^{\mu}_{\varphi\to K^+(p_+)K^-(p_-)} &= -\mathrm{i}\,g_{\varphi KK}(p_+ + p_-)^\mu,\\
\Gamma_{f_0\to K^+(p_+)K^-(p_-)} &= -\mathrm{i}\,g_{f_0KK},
\end{aligned}$$

where

$$g_{\varphi KK} = -\frac{g_\rho}{\sqrt{2}},$$

$$g_{f_0 KK} = 2Z_K\left(\sqrt{2}g_{\sigma_s}\cos(\bar{\varphi})(2m_s - m_u) - g_{\sigma_u}\sin(\bar{\varphi})(2m_u - m_s)\right) \approx 6.14\,\mathrm{GeV},$$

where $m_u = 0.263$ GeV is the up quark mass $Z_K \approx Z_\pi = \left(1 - 6m_u^2/M_{a_1}^2\right)^{-1} = 1.378$ and $M_{a_1} = 1.23$ GeV.

The amplitude of kaon–loop contribution has the form

$$M^{(K)}(\gamma^*(q_1,\mu) \to \varphi(q_2,\nu) f_0(q_3)) = \frac{\mathrm{i}\,e}{(2\pi)^4} g_{\varphi KK} g_{f_0 KK} \varepsilon_\mu(q_1)\varepsilon_\nu(q_2) \times \int \mathrm{d}k \frac{(2k - q_+ + q_-)^\mu (2k - q_+)^\nu}{\left((k-q_2)^2 - M_K^2\right)(k^2 - M_K^2)\left((k+q_1)^2 - M_K^2\right)}, \tag{6}$$

where $M_K = 0.493$ GeV is the mass of $K^\pm$ mesons. Integrating over loop momentum in (6) one gets the amplitude contribution to the form (1), where

$$A_{(K)} = \int_0^1 \mathrm{d}x \int_0^1 \frac{y\mathrm{d}y}{D_K}\left(-\alpha_K + \frac{q_1^2 q_2^2}{(q_1 q_2)^2}\beta_K\right), \; B_{(K)} = \int_0^1 \mathrm{d}x \int_0^1 \frac{y\mathrm{d}y}{D_K}\beta_K, \; C_{(K)} = e\, g_{\varphi KK}\, g_{f_0 KK}$$

where $\alpha_K = -4xy\bar{y}$, $\beta_K = 4y(xy - \frac{1}{2})$, $D_K = M_K^2 + q_x^2 y^2 - y h_x$.

2.3. *Process* $e^+e^- \to \gamma^* \to \varphi f_0(980)$

The matrix element of this process has the form similar to (1):

$$M\left(e^+(p_+)e^-(p_-) \to \gamma^*(q_1) \to \varphi(q_2) f_0(q_3)\right) = \frac{4\pi\alpha}{s} J_\mu^{\mathrm{QED}} \sum_{i=q,K} C_{(i)}\varepsilon_\nu(q_2)\left(A_{(i)}\, R_{\mu\nu}^{(1)} + B_{(i)}\, R_{\mu\nu}^{(2)}\right), \tag{7}$$

where $J_\mu^{\mathrm{QED}} = \bar{v}(p_+)\gamma_\mu u(p_-)$ is the electromagnetic current of electron and positron annihilation ($J_\mu q_1^\mu = 0$), and $e_\nu(q_2)$ is the polarization 4-vector of φ-meson ($q_{2\nu} e_\nu(q_2) = 0$).

Summation over polarization states of matrix element square gives

$$\sum_{pol} |M|^2 = \left(\frac{8\pi\alpha}{s}\right)^2 \left\{ s_1^2 |A|^2 - \frac{1}{2}\left(|A - \tilde{B}|^2 s - |\tilde{B}|^2 \frac{s_1^2}{4M_\varphi^2}\right)\left(E_2^2\left(1 - \beta_2^2 c^2\right) - M_\varphi^2\right)\right\}, \tag{8}$$

where $s_1 = 2(q_1 q_2) = s + M_\varphi^2 - M_{f_0}^2$, $\tilde{B} = (1/s_1^2)B(4sM_\varphi^2)$, $E_2 = [1/(2\sqrt{2})](s + M_\varphi^2 - M_{f_0}^2)$ is φ-meson c.m.s. energy, $c = \cos\theta = \cos(\boldsymbol{p}_-, \boldsymbol{q}_2)$ is the cosine of emission angle of φ meson and $\beta_2^2 = \lambda(s, M_\varphi^2, M_f^2)/[(s + M_\varphi^2 - M_f^2)^2]$ is the velocity squared of the φ meson. $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ is the well-known triangle function. The quantities A and B are defined as

$$\begin{aligned} A &= C_{(q)} A_{(q)} + C_{(K)} A_{(K)}, \\ B &= C_{(q)} B_{(q)} + C_{(K)} B_{(K)}. \end{aligned}$$

The phase volume is

$$\mathrm{d}\Gamma = \frac{\mathrm{d}^3 q_2}{2E_2} \frac{\mathrm{d}^3 q_3}{2E_3} \frac{(2\pi)^4}{(2\pi)^6} \delta^4(p_+ + p_- - q_2 - q_3) = \frac{\sqrt{\lambda(s, M_\varphi^2, M_{f_0}^2)}}{16\pi s} \, \mathrm{d}\cos\theta \,. \quad (9)$$

Then the differential cross section can be written in the form

$$\frac{\mathrm{d}\sigma^{e^+e^- \to \varphi f_0}}{\mathrm{d}\cos\theta} = \frac{\pi\alpha^2}{s} \left(D(s) + E(s) \cos^2\theta \right). \quad (10)$$

3. Conclusion

One can see that both contributions of quark loop and kaon loop are large and almost completely compensate each other (destructive interference). The total cross section dependence on total energy s is given in Fig. 1. We obtained the result which is very promising for the upcoming experiment in C-τ factory in China.

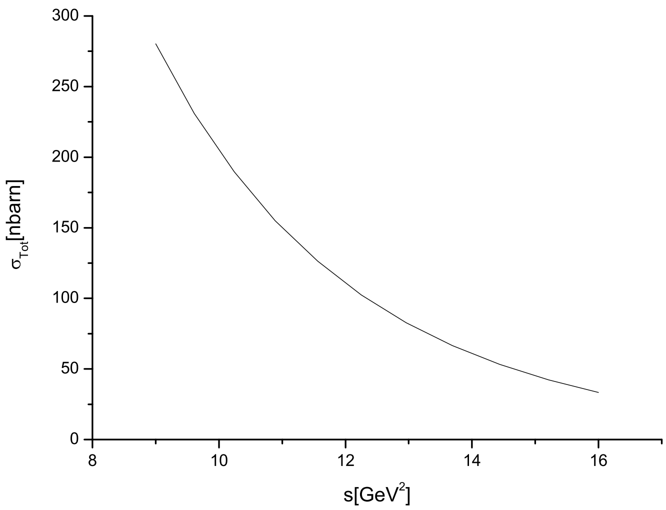

Fig. 1. Dependence of the total cross section on the total energy s.

Acknowledgements

One of us (E.K.) is grateful to the Institute of Physics, SAS. The work was partly supported also by the Slovak Grant Agency for Sciences VEGA, Grant No. 2/7116/27 and the grant RFFI 05-02-16699.

References

[1] N. N. Achasov and A. V. Kiselev, Phys. Rev. D **68** (2003) 014006.

[2] N. N. Achasov and G. N. Shestakov, Phys. Rev. Lett. **92** (2004) 182001.

[3] N. N. Achasov, S. A. Devianin and G. N. Shestakov, Sov. Phys. Usp. **27** (1984) 161.

[4] L. Montanet, Rep. Prog. Phys. **46** (1983) 337.

[5] N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. B **315** (1989) 465.

[6] M. N. Achasov et al., Phys. Lett. B **438** (1998) 441.

[7] KLOE Collaboration, A. Aloisio et al., Phys. Lett. B **536** (2002) 209.

[8] S. Ivashyn and A. Y. Korchin, Eur. Phys. J. C **54** (2008) 89; arXiv:0707.2700 [hep-ph].

[9] M. N. Achasov et al., Phys. Lett. B **479** (2000) 53; F. Ambrosino et al. [KLOE Collaboration], Phys. Lett. B **634** (2006) 148.

TVORBA f_0-MEZONA PROCESOM $e^+e^- \to \varphi f_0$

Zagonetna priroda mezona $f_0(980)$ može se istraživati u tvornicama C–τ s ukupnom energijom e^+e^- u sustavu središta mase $3 < \sqrt{s} < 4$ GeV. Takav pogon ima prednost prema tvornicama φ–mezona sa $\sqrt{s} = 1.02$ GeV u kojima se može istraživati proces $e^+e^- \to \gamma f_0(980)$. Uzrok je snažno potisnuće udarnog presjeka faktorom $\sim \omega^3$ zbog faznog prostora u mjerenjima u tvornicama φ. Naš se pristup zasniva na analizi Feynmanovih diagrama petlja najnižeg reda uzimajući strani sastavni kvark ili nabijeni kaon kao virtualne čestice u petlji.