ISSN 1330–0016
CODEN FIZBE7

CHROMODYNAMIC LENSING AND $\perp$ SINGLE SPIN ASYMMETRIES

M. BURKARDT

Department of Physics, New Mexico State University, Las Cruces, NM 88003, U.S.A.

Received 4 September 2003; Accepted 30 August 2004
Online 22 November 2004

The physical interpretation of generalized parton distributions (in the limit $\xi = 0$) as Fourier transforms of impact parameter dependent parton distributions is discussed. Particular emphasis is put on the role of the target polarization. For transversely polarized targets, we expect a significant deviation from axial symmetry for the distribution in the transverse plane. We conjecture that this transverse distortion, in combination with the final state, provides a natural explanation for the sign of the Sivers contribution to semi-inclusive single-spin asymmetries.

PACS numbers: 12.38Lg 13.85Ni UDC 539.12
Keywords: Sivers effect, generalized parton distributions

1. Introduction

For many years, deep inelastic scattering (DIS) experiments have been a useful tool for exploring hadron structure. In the Bjorken limit, these experiments probe the parton distributions $q(x)$ which can be defined as the expectation value of a light-like correlation function

$$q(x) = \int \frac{\mathrm{d}x^-}{2\pi} \, \langle\, p \,|\, \bar{q}\left(-\frac{x^-}{2}, \mathbf{0}_\perp\right) \gamma^+ q\left(\frac{x^-}{2}, \mathbf{0}_\perp\right) |\, p \,\rangle \, \mathrm{e}^{\mathrm{i}x^- x_{\mathrm{Bj}} P^+}. \tag{1}$$

Throughout this paper, we use light-cone variables $x^\pm = \frac{1}{\sqrt{2}}\left(x^0 \pm x^3\right)$.

The physical interpretation of $q(x)$ is that of a light-cone momentum density of quarks in the target, where x is the fraction of the target's light-cone momentum that is carried by the active quark. However, Eq. (1) provides no information about the position of the quarks.

A generalization of Eq. (1) to non-forward matrix elements yields the generalized parton distributions (GPDs) [1−3][1]

$$GPD(x,\xi,t) \equiv \int \frac{\mathrm{d}x^-}{2\pi} \,\langle\, p' \,|\, \bar{q}\left(-\frac{x^-}{2},\mathbf{0}_\perp\right)\gamma^+ q\left(\frac{x^-}{2},\mathbf{0}_\perp\right) |\, p \,\rangle\, \mathrm{e}^{\mathrm{i}x^- xP^+} \tag{2}$$

with $\Delta = p - p'$, $t = \Delta^2$, and $\xi(p^+ + p^{+\prime}) = -2\Delta^+$. Experimentally, these GPDs can, for example, be probed in deeply virtual Compton scattering.

Recently, people became very interested in GPDs after it became clear that they can be linked to a number of other observables. For example, upon integration over x, they can be related to form factors. In that sense, they provide a decomposition of form factors w.r.t. the (average) light-cone momentum of the active quark. For example, such information can be very useful to understand the form-factors mechanisms at high momentum transfer. Another application of GPDs is that knowing GPDs would enable us to determine a quantity that can be identified with the total (spin + orbital) angular momentum carried by the quarks in the nucleon. If fact, it becomes more and more clear that GPDs could provide us with key information about the orbital angular momentum structure of the nucleon.

However, there is another, very interesting piece of information about the structure of hadrons that GPDs can provide, namely, they can teach us how partons are distributed in the transverse plane. Discussing this connection and possible consequences will be the main purpose of this work.

2. Impact-parameter dependent parton distributions

In order to help us understand the kind of information that is contained in GPDs, in simple physical pictures, we explore the analogy to form factors. Indeed, we can write the definition of the GPDs H and E in a form that emphasizes this analogy

$$\left\langle p' \left| \hat{O} \right| p \right\rangle = H(x,\xi,\Delta^2)\, \bar{u}(p')\, \gamma^+ u(p) + E(x,\xi,\Delta^2)\, \bar{u}(p')\, \frac{\mathrm{i}\sigma^{+\nu}\Delta_\nu}{2M}\, u(p) \tag{3}$$

with $\hat{O} \equiv \int \mathrm{d}x^- \,(\frac{1}{2\pi})\, \exp(\mathrm{i}x^- \bar{p}^+ x)\; \bar{q}(-x^-/2)\; \gamma^+ q(x^-/2)$. The only difference between Eq. (3) and the definition of the Dirac and Pauli form factors F_1 and F_2 is the fact that the current density operator is substituted by the operator $\hat{O}$. When sandwiched between momentum eigenstates, this operator $\hat{O}$ acts like a "filter" that lets through only quarks that carry a certain momentum fraction x, and GPDs are the form factors of this momentum filter.

The forward matrix element of the vector current gives the charge. Form factors are the non-forward matrix element of the vector current operator. By taking the Fourier transform of the form factor, we can learn how the charge (i.e. the physical quantity that is related to the forward matrix element) is distributed in position space.

[1] For a recent comprehensive review see Ref. [4].

Operator	Forward matrix elem.	Off-forward matrix elem.	Position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\boldsymbol{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi}\, \bar{q}\left(\frac{-x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,0,t)$	$q(x,\mathbf{b}_\perp)$

Fig. 1. Illustration of the analogy between the form factor ↔ charge distribution correspondence and the GPD ↔ impact parameter dependent parton distribution correspondence.

In the case of GPDs, the forward matrix element gives the usual parton distribution functions (PDFs). By analogy to the form factor of the vector current, one would therefore expect that some Fourier transform of GPDs provides information about how the usual PDFs are distributed in position space (Fig. 1) Working out the details of this will be the subject of the first part of this work.

Of course, since the usual PDFs already measure the longitudinal momentum of the quarks, Heisenberg's uncertainty principle allows us to measure only the transverse position of the partons. Because of that, we will in the following only consider the case when the momentum transfer in GPDs is purely transverse (i.e. $\xi \propto \Delta^+ = 0$).[2]

Before we proceed and derive the connection between GPDs and PDFs in transverse position ("impact parameter") space, we need to define what we mean by impact-parameter-dependent PDFs. For this purpose we introduce wave packets that have a sharp longituninal momentum and that are localized in transverse position [6−8]

$$\left| p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda \right\rangle \equiv \mathcal{N} \int \mathrm{d}^2\mathbf{p}_\perp \left| p^+, \mathbf{p}_\perp, \lambda \right\rangle , \tag{4}$$

where $\mathcal{N}$ is a normalization constant, such that $(2\pi)^2 \int \mathrm{d}^2\mathbf{p}_\perp \left|\mathcal{N}\right|^2 = 1$. This state is localized in impact parameter space in the sense that it is an eigenstate of the $\perp$ center of (longitudinal) momentum

$$\mathbf{R}_\perp \equiv \frac{1}{p^+} \int \mathrm{d}x^- \mathrm{d}^2\mathbf{x}_\perp \, T^{++}(x)\mathbf{x}_\perp \, , \tag{5}$$

where T^{++} is the component of the energy momentum tensor that describes the light-cone momentum density. The parton representation for the $\perp$ center of momentum is the weighted average of $\perp$ parton positions, where the weight factors are the fractions of p^+ momentum carried by each parton, i.e. $\mathbf{R}_\perp = \sum_i x_i \mathbf{r}_{\perp i}$. Working with this transversely localized state is in many ways analogous to working in the center of mass frame in nonrelativistic systems.

[2]Note that if one makes only an approximate measurement of the longitudinal momentum, then one can still make an (approximate) measurement of the longitudinal position, as long as the Heisenberg inequality is obeyed [5].

Using this state, we can now define what we mean by **impact-parameter-dependent parton distributions**. For example, for the unpolarized distributions, we define [9]

$$q(x,\mathbf{b}_\perp) \equiv \int \frac{\mathrm{d}x^-}{4\pi} \left\langle p^+,\mathbf{0}_\perp \right| \bar{q}\left(-\frac{x^-}{2},\mathbf{b}_\perp\right) \gamma^+ q\left(\frac{x^-}{2},\mathbf{b}_\perp\right) \left| p^+,\mathbf{0}_\perp \right\rangle \mathrm{e}^{\mathrm{i}xp^+x^-}. \tag{6}$$

In gauges other than light-cone gauge, a straight-line gauge string needs to be included in Eq. (6). A very similar definition can be given for the polarized impact parameter dependent parton distribution $\Delta q(x,\mathbf{b}_\perp)$.

Using translation invariance, it is straightforward to relate $q(x,\mathbf{b}_\perp)$ to GPDs

$$\begin{aligned}
q(x,\mathbf{b}_\perp) &\equiv \int\mathrm{d}x^- \left\langle p^+,\mathbf{R}_\perp=\mathbf{0}_\perp \right| \bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp) \left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right\rangle \mathrm{e}^{\mathrm{i}xp^+x^-} \\
&= |\mathcal{N}|^2 \int\mathrm{d}^2\mathbf{p}_\perp \int\mathrm{d}^2\mathbf{p}'_\perp \int\mathrm{d}x^- \left\langle p^+,\mathbf{p}'_\perp \right| \bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp) \left|p^+,\mathbf{p}_\perp\right\rangle \mathrm{e}^{\mathrm{i}xp^+x^-} \\
&= |\mathcal{N}|^2 \int\mathrm{d}^2\mathbf{p}_\perp \int\mathrm{d}^2\mathbf{p}'_\perp \int\mathrm{d}x^- \left\langle p^+,\mathbf{p}'_\perp \right| \bar{q}(-\frac{x^-}{2},\mathbf{0}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{0}_\perp) \left|p^+,\mathbf{p}_\perp\right\rangle \mathrm{e}^{\mathrm{i}xp^+x^-} \\
&\qquad\qquad \times\, \mathrm{e}^{\mathrm{i}\mathbf{b}_\perp\cdot(\mathbf{p}_\perp-\mathbf{p}'_\perp)} \\
&= |\mathcal{N}|^2 \int\mathrm{d}^2\mathbf{p}_\perp \int\mathrm{d}^2\mathbf{p}'_\perp H\left(x,0,-\left(\mathbf{p}'_\perp-\mathbf{p}_\perp\right)^2\right) \mathrm{e}^{\mathrm{i}\mathbf{b}_\perp\cdot(\mathbf{p}_\perp-\mathbf{p}'_\perp)}
\end{aligned} \tag{7}$$

Upon switching variables to sums and differences of momenta, one finds that the GPD $H(x,0,-\mathbf{\Delta}_\perp^2)$ is the Fourier transform of $q(x,\mathbf{b}_\perp)$ [7]

$$q(x,\mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\mathbf{\Delta}_\perp}{(2\pi)^2} H(x,0,-\mathbf{\Delta}_\perp^2)\, \mathrm{e}^{\mathrm{i}\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}. \tag{8}$$

Besides being the Fourier transform of GPDs, $q(x,\mathbf{b}_\perp)$ satisfies a number of positivity constraints. For example [9],

$$\begin{aligned} q(x,\mathbf{b}_\perp) &> 0 \quad \text{for} \quad x>0\,, \\ q(x,\mathbf{b}_\perp) &< 0 \quad \text{for} \quad x<0\,, \end{aligned} \tag{9}$$

where the minus sign for $x<0$ follows from charge conjugation. The proof of these positivity constraints parallels the proof that the usual PDFs are positive. As a result, one can also derive various "Soffer-type" inequalities among PDFs in impact parameter space [10].

For the practitioner, positivity constraints are useful because they provide model-independent theoretical constraints on any phenomenological ansatz for GPDs. However, a much more important consequence of these inequalities is that they allow a probabilistic interpretation for $q(x,\mathbf{b}_\perp)$, which indicates that $q(x,\mathbf{b}_\perp)$ has a physical meaning above and beyond being the Fourier transform of $H(x,0,-\mathbf{\Delta}_\perp^2)$.

2.1. Discussion

Knowledge of GPDs for purely transverse momentum transfer allows probing parton distributions in impact parameter space. This is completely novel information about the nucleon structure and will provide interesting tests for our understanding of the quark-gluon structure of hadrons.

The reference point for the impact parameter dependent PDFs is the $\perp$ center of momentum $\mathbf{R}_\perp \equiv \sum_{i=q,g} x_i \mathbf{r}_{\perp,i}$. When $x \to 1$, the active quark becomes the center of momentum and as a result the transverse width of $b(x, \mathbf{b}_\perp)$ should go to zero. Note that this does not mean that the transverse size of the nucleon goes to zero, since for example the distance $\mathbf{B}_\perp$ between the active quark and the center of momentum of the spectators can remain finite as $x \to 1$, since $\mathbf{b}_\perp = (1-x)\mathbf{B}_\perp$. However, what the vanishing $\perp$ width implies is that $H(x, 0, t)$ should become t-independent as $x \to 1$.[3] For decreasing x, one expects the size of the nucleon to grow, because when $x \sim m_\pi/M$, one should see the pion cloud, and for even smaller x, a logarithmic growth of the $\perp$ size with $1/x$ should set in [11]. Of course, when x decreases, not only the width of $q(x, \mathbf{b}_\perp)$ should increase but also its magnitude since there are more quarks at small x. In order to gain some intuitive understanding about this behavior, we show $q(x, \mathbf{b}_\perp)$ in Fig. 2 for a simple model that has all these features built in.

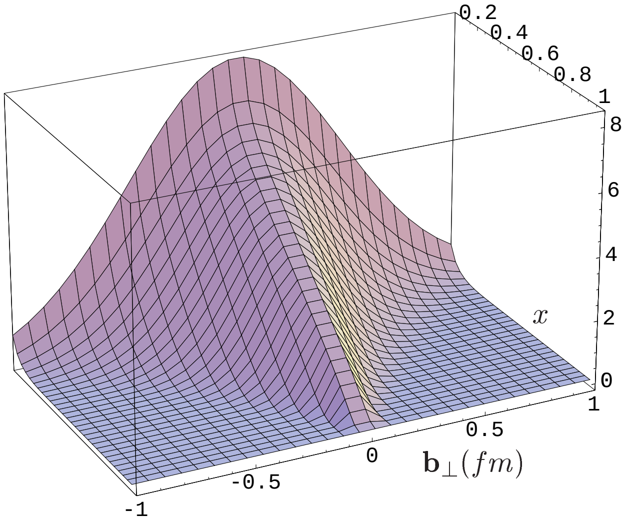

Fig. 2. Anticipated shape of $q(x, \mathbf{b}_\perp)$ (qualitative).

Note that deeply-virtual Compton scattering (DVCS) experiments probe always $\Delta^+ = 0$. Therefore, from the point of view of DVCS, the limit $\Delta^+ = 0$ is unphysical and can only be reached by extrapolation. However, this task is facilitated by the fact that the x-moments of $E(x, \xi, t)$ have a polynomial dependence on ξ and

[3] Of course, at the same time $H(x, 0, t)$ should go to zero since $q(x)$ goes to zero as $x \to 1$ and, therefore, whether or not the form factor receives a significant contribution from $x \to 1$ ("Feynman mechanism") depends on details.

therefore, at least theoretically, the extrapolation to $\xi = 0$ can be done model independently.

So far we did not discuss the scale-dependence of GPDs. Adding scale dependence to our above results is trivial since QCD evolution, which addresses only the divergent part of the Q^2 dependence, is local in impact parameter space, i.e. there is no mixing between different $\mathbf{b}_\perp$, and there are separate DGLAP evolution equations for each $\mathbf{b}_\perp$. This is consistent with the fact that the evolution of GPDs is t-independent. Of course, this is valid only for $Q^2 \gg t$ and therefore $1/Q^2$ limits the transverse "pixel size" in $q(x, \mathbf{b}_\perp)$.

3. The physics of $E(x, 0, -\mathbf{\Delta}_\perp^2)$

For $\Delta^+ = 0$, the GPD $E(x, 0, -\mathbf{\Delta}_\perp^2)$ only contributes to helicity flip amplitudes

$$\begin{aligned} \int \frac{\mathrm{d}x^-}{4\pi} \mathrm{e}^{\mathrm{i}p^+x^-x} \left\langle P{+}\Delta,\uparrow \left| \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| P,\uparrow \right\rangle &= H(x,0,-\mathbf{\Delta}_\perp^2)\,, \qquad (10) \\ \int \frac{\mathrm{d}x^-}{4\pi} \mathrm{e}^{\mathrm{i}p^+x^-x} \left\langle P{+}\Delta,\uparrow \left| \bar{q}\left(\frac{-x^-}{2}\right) \gamma^+ q\left(\frac{x^-}{2}\right) \right| P,\downarrow \right\rangle &= -\frac{\Delta_x - \mathrm{i}\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2)\,. \end{aligned}$$

Therefore, in order to understand the physics of $E(x, 0, -\mathbf{\Delta}_\perp^2)$, we need to consider states that are not helicity eigenstates. The contribution from E is maximal in states that have equal probability from both helicities, and we therefore consider the state

$$|X\rangle \equiv \frac{1}{\sqrt{2}} \left[\left| p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow \right\rangle + \left| p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow \right\rangle \right], \qquad (11)$$

which one may interpret as a state that has a transverse center of momentum localized at the origin and that is polarized in the $\hat{x}$ direction in the infinite momentum frame.[4] We denote the *unpolarized* quark distribution in impact parameter space for this transversely polarized state by $q_X(x, \mathbf{b}_\perp)$, i.e.

$$q_X(x, \mathbf{b}_\perp) \equiv \int \frac{\mathrm{d}x^-}{4\pi} \langle X | \, \bar{q}\left(-\frac{x^-}{2}, \mathbf{b}_\perp\right) \gamma^+ q\left(\frac{x^-}{2}, \mathbf{b}_\perp\right) |X\rangle \, \mathrm{e}^{\mathrm{i}xp^+x^-}. \qquad (12)$$

In order to relate $q_X(x, \mathbf{b}_\perp)$ to GPDs, we follow the same steps as in Eq. (7). The only difference is that one now obtains both matrix elements that are diagonal in the target spin, as well as matrix elements that involve a target spin flip. Making use of Eq. (10), one thus finds after some integration by parts that the unpolarized quark distribution in a transversely polarized nucleon is the same as the unpolarized quark distribution in a longitudinally polarized (or unpolarized) nucleon plus a correction

[4] The rest frame interpretation of the state may be subject to Wigner-Melosh rotations, i.e. when viewed from the rest frame, this state corresponds to a nucleon polarized in the x direction plus some relativistic corrections.

term. The correction term is proportional to the gradient of the Fourier transform of $E(x, 0, -\mathbf{\Delta}_\perp^2)$

$$q_X(x,\mathbf{b}_\perp) = q(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{\mathrm{d}^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x,0,-\mathbf{\Delta}_\perp^2)\, \mathrm{e}^{\mathrm{i}\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}. \tag{13}$$

If the nucleon is longitudinally polarized, then rotational symmetry around the z-axis implies that the impact-parameter-dependent PDF $q(x, \mathbf{b}_\perp)$ is axially symmetric (depends only on $\mathbf{b}_\perp^2$). However, when the nucleon is transversely polarized, then there is no reason why $q_X(x, \mathbf{b}_\perp)$ should be axially symmetric. The distortion is described by the gradient of the Fourier transform of $E(x, 0, -\mathbf{\Delta}_\perp^2)$.

The direction of the distortion can be easily understood from a simple classical picture: In DIS one probes the + component of the current. Since $j^+ = j^0 + j^z$, the distortion arises because an orbital motion around the x-axis produces a j^z-current that is asymmetric w.r.t. $\pm\hat{y}$. This explains from an intuitive point of view why the $\hat{y}$ derivative appears in Eq. (13).

In order to understand the magnitude of the distortion, we would have to know the function $E(x, 0, -\mathbf{\Delta}_\perp^2)$. However, even without knowing $E(x, 0, -\mathbf{\Delta}_\perp^2)$, we can still estimate the mean effect by evaluating the transverse flavor dipole moment that results from this distortion

$$d_y^q \equiv \int \mathrm{d}x \int \mathrm{d}^2\mathbf{b}_\perp q_X(x, \mathbf{b}_\perp) b_y = \frac{1}{2M}\int \mathrm{d}x E_q(x, 0, 0) = \frac{1}{2M} F_{2q}(0) = \frac{\kappa_q^p}{2M}\,, \tag{14}$$

where κ_q^p is the anomalous magnetic moment contribution to the proton from flavor q. A simple $SU(3)$ analysis (neglecting the small strange quark contribution[5]) yields $\kappa_u^p \approx 1.67$ and $\kappa_d^p \approx -2.03$, i.e., the resulting flavor dipole moments are on the order of $0.1 - 0.2$ fm, which is a significant effect.

In order to illustrate the magnitude of the anticipated distortion, we take the model for $H_q(x, 0, t)$ that was used in Fig. 1, and as a model for $E_q(x, 0, t)$, we make the ansatz

$$E_u(x, 0, t) = \frac{1}{2}\kappa_u H_u(x, 0, t)\,, \qquad E_d(x, 0, t) = \kappa_d H_d(x, 0, t)\,. \tag{15}$$

The factor $\frac{1}{2}$ accounts for the fact that $H_u = 2H_d$ in this very simple model. Of course, we do not really expect that H and E are proportional. But for the purpose of providing a rough picture of the expected effects, this crude ansatz may be useful. The resulting parton distributions in impact parameter space are shown in Fig. 3 Even though any details (e.g. x-dependence) of the distortion are, of course, model dependent, the mean magnitude of the effect is constrained by Eq. (14) and thus model-independent. Figure 3 thus clearly illustrates that the anticipated distortion is quite significant. Notice that the opposite signs of the distortion for u and d quarks are due to the fact that κ_u and κ_d have opposite signs. The fact that the

[5]Note that $\kappa_u^p - \kappa_d^p = \kappa^p - \kappa^n \approx 3.7$ is independent of the strange magnetic moment.

distortion is larger for d than for u quarks is due to the fact that E_u and E_d have about the same magnitude, but H_u is twice as large as H_d.

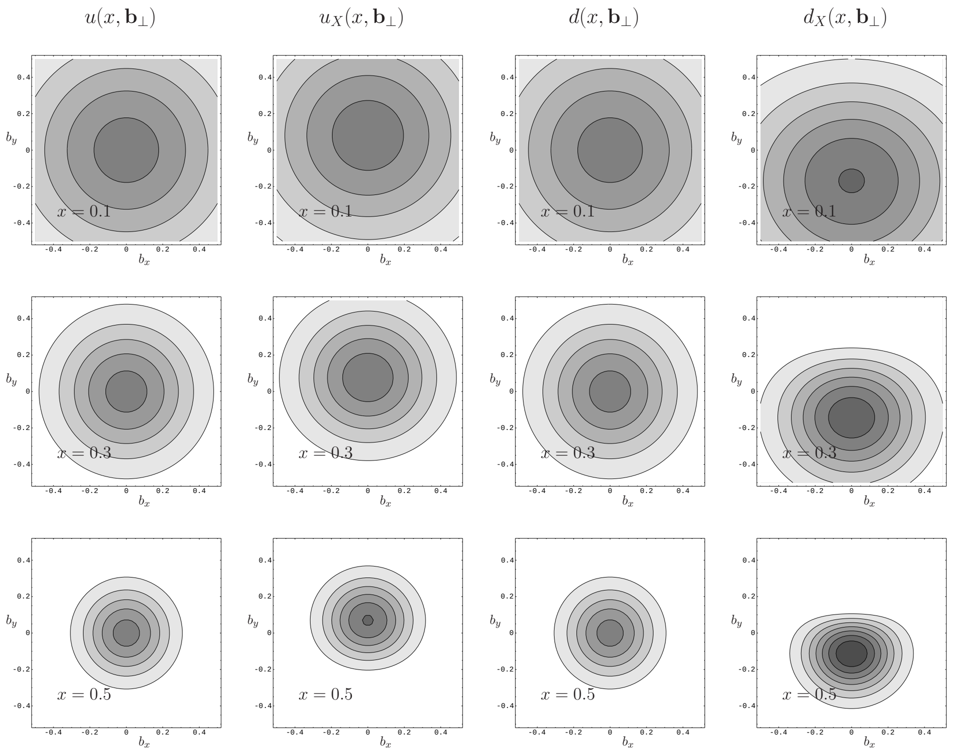

Fig. 3. Parton distributions in impact parameter space for the simple model for $x = 0.1, 0.3, 0.5$. For each plot the grayscales are normalized to the central value. 1^{st} column: (unpolarized) distribution of u quarks in a longitudinally polarized nucleon. 2^{nd} column: same for a nucleon that is polarized in the $\hat{x}$ direction. 3^{rd} and 4^{th} columns: same for d quarks.

4. $\perp$ *single spin asymmetries*

In the previous section, we demonstrated that quark distribution functions in a transversely polarized nucleon are expected to have a significant left-right (w.r.t. the spin) asymmetry in impact-parameter space. In this section we would like to add some speculations about possible ramifications of this effect for other experiments. In particular, we will focus on the transverse single spin asymmetry in semi-inclusive photo-production of mesons off a transversely polarized target.

For example, let us consider the inclusive production of π^+ and π^0 mesons off nucleons that are polarized into the plane, with unpolarized photons coming in from the $-\hat{z}$-direction. Since $e_u^2 = 4e_d^2$, and since $u \to \pi^+, \pi^0$ fragmentation is 'favored', most π^+, π^0 mesons result from an initial up quark that has been knocked out. At the quark level, several mechanisms have been proposed that can give rise to a left-right asymmetry (relative to the nucleon spin) of the produced mesons. In the Collins effect, a transversely polarized quark fragments with a left-right asymmetry into mesons. Here we are not discussing this effect. Instead, we discuss the Sivers effect where the outgoing u quark has already a left-right asymmetry *before* it fragments. Of course, a left-right asymmetry in the momentum $\mathbf{k_q}$ of a quark in the nucleon is inconsistent with time-reversal invariance [$\mathbf{k_q} \cdot (\mathbf{S}_N \times \mathbf{p_N})$ is T-odd], and therefore any $\mathbf{k_q}$ asymmetry can only arise from the final-state interactions (FSI) of the struck quark as it escapes from the target. The FSI can be conveniently included in a gauge invariant-definition of unintegrated parton densities [12, 13]

$$P(x, \mathbf{k}_\perp, \mathbf{s}_\perp) = \int \frac{\mathrm{d}y^- \mathrm{d}^2\mathbf{y}_\perp}{16\pi^3} \mathrm{e}^{-\mathrm{i}xp^+y^- + \mathrm{i}\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \left\langle p \left| \bar{q}(0, y^-, \mathbf{y}_\perp) W_{y\infty}^\dagger \gamma^+ W_{0\infty} q(0) \right| p \right\rangle \quad (16)$$

$W_{y\infty} = P \exp\Big(-\mathrm{i}g \int\limits_{y^-}^{\infty} \mathrm{d}z^- A^+(y^+, z^-, \mathbf{y}_\perp) \Big)$ indicates a path ordered Wilson-line operator going out from the point y to infinity. Starting from Eq. (16), one finds for the mean transverse momentum [14]

$$\langle \mathbf{k}_x \rangle = \int\limits_0^\infty \mathrm{d}y^- \left\langle p, s \left| \bar{q}(0, 0^-, \mathbf{0}_\perp) W_{0y}^\dagger \gamma^+ G^{+x}(y^-, \mathbf{0}_\perp) W_{0y} q(0, 0^-, \mathbf{0}_\perp) \right| p, s \right\rangle , \quad (17)$$

where $G^{\mu\nu}$ is the QCD field strength tensor. Apart from the gauge link factors W_{0y}, which are only there to make Eq. (17) gauge invariant, this result has a very simple physical interpretation: The mean transverse momentum of the outgoing quark is obtained by integrating the transverse force (from G^{+x}) along its outward path (which is along the light-cone for a high-energy process). Although Eq. (17) (without the gauge links) has been written down a long time ago [15], the momentum space expression has not helped much to estimate the size, or even the sign, of $\langle k_x \rangle$ in QCD. As an application of the impact parameter picture, we will attempt in the following to predict the sign of the Sivers asymmetry.

For this purpose, we first make use of Galilei invariance under $\perp$ boosts to rewrite Eq. (17) in impact parameter space [16]

$$\begin{aligned} \langle \mathbf{k}_x \rangle = \int\limits_0^\infty \mathrm{d}y^- \int \mathrm{d}^2\mathbf{b}_\perp \Big\langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, s \Big| \bar{q}(0, 0^-, \mathbf{b}_\perp) W_{0^-\mathbf{b}_\perp, y^-\mathbf{b}_\perp}^\dagger \gamma^+ \qquad (18) \\ \times\ G^{+x}(y^-, \mathbf{b}_\perp) W_{0^-\mathbf{b}_\perp, y^-\mathbf{b}_\perp} q(0, 0^-, \mathbf{b}_\perp) \Big| p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, s \Big\rangle . \end{aligned}$$

The r.h.s. of Eq. (18) can be interpreted as the correlation between the transverse position of the quark and the transverse impulse that the quark experiences from the FSI, when it is knocked out from that transverse position. Intuitively, we would expect the FSI on average to be attractive, since it costs energy to build up the 'string' of gauge fields that connects the escaping quark with the spectators before quark pair creation leads to a breaking of this 'string'. Although the actual force that acts on the struck quark is a complicated superposition of forces from all spectators, we still expect that the average force still has some of the features of this semi-classical string picture, and hence we expect (on average) an attractive force on the outgoing quark. We should emphasize that many phenomenological models [17, 18] have this feature implicitly built in.

Consider now a photon which is moving in the $-\hat{z}$-direction, that collides with a nucleon that is polarized in the $+\hat{y}$-direction. According to the results of the previous section, when viewed from the Breit frame, the u quarks tend to be displaced in the $-\hat{x}$-direction in impact-parameter space. If, as we argued above, there is on average an attractive force on the u quark after it has been struck by the photon, then that force should have a component in the $+\hat{x}$-direction. Therefore, we expect that $\langle k_x \rangle > 0$. If we reverse the nucleon spin, then the distortion in transverse position space gets reversed and $\langle k_x \rangle$ changes sign, as it should be. Explicit model calculations [17, 18] confirm these results. However, we should emphasize that our results for the Sivers asymmetry are model independent in the sense that we do not specify details of the FSI — we only postulate that they are on average attractive (towards the spectators).

Another model-independent result that we have derived is that the sign of the Sivers asymmetry is essentially[6] determined if one knows the sign of the anomalous magnetic moments contribution from a given quark flavor and the sign (attractive or repulsive) of the FSI. A similar correlation has been observed in Ref. [12]. As a result, we expect for example that the Sivers asymmetries for u and d quarks have opposite signs.

Acknowledgements

I would like to thank X. Ji and N. Makins for very stimulating discussions. This work was supported in part by the DOE under grant number DE-FG03-95ER40965.

References

[1] D. Müller et al., Fortschr. Phys. **42** (1994) 101.

[2] X. Ji, Phys. Rev. Lett. **78** (1997) 610.

[3] A. Radyushkin, Phys. Rev. D **58** (1998) 114008.

[4] M. Diehl, Phys. Rept. **388** (2003) 41.

[5] X. Ji, Phys. Rev. Lett. **91** (2003) 062001.

[6] D. E. Soper, Phys. Rev. D **15** (1977) 1141.

[7] M. Burkardt, Phys. Rev. D **62** (2000) 071503; Erratum-ibid. **66** (2002) 119903.

[6] Here one assumes only that these functions don't have fluctuating signs.

[8] M. Diehl, Eur. Phys. J. **C25** (2002) 223.

[9] M. Burkardt, in *Lepton Scattering, Hadrons and QCD*, Adelaide, Australia, March 2001, hep-ph/0105324.

[10] P. V. Pobylitsa, Phys. Rev. D **67** (2003) 034009.

[11] M. Strikman and C. Weiss, Phys. Rev. D **69** (2004) 054012.

[12] S. J. Brodksy, D. S. Hwang and I. Schmidt, Phys. Lett. B **530** (2002) 99.

[13] J. C. Collins, Phys. Lett. B **536** (2002) 43; X. Ji and F. Yuan, Phys. Lett. B **543** (2002) 66; A. Belitsky, X. Ji and F. Yuan, Nucl. Phys. B **656** (2003) 165.

[14] D. Boer, P. J. Mulders, and F. Pijlman, Nucl. Phys. **B667** (2003) 201.

[15] A. Schäfer and L. Mankiewicz, Phys. Rev. D **47** (1993) R1; J. Qiu and G. Sterman, Phys. Rev. D **59** (1999) 014004.

[16] M. Burkardt, Phys. Rev. D **66** (2002) 114005; M. Burkardt, Nucl. Phys. A **735** (2004) 185.

[17] F. Yuan, Phys. Lett. B **575** (2003) 45.

[18] M. Burkardt and D. S. Hwang, Phys. Rev. D **69** (2004) 074032.

KROMODINAMIČKO FOKUSIRANJE I JEDNOSPINSKE ⊥ ASIMETRIJE

Raspravlja se fizičko objašnjenje poopćenih partonskih raspodjela (u granici $\xi = 0$) kao Fourierovih preobrazbi partonskih raspodjela ovisnih o parametru sudara. Poseban se naglasak stavlja na ulogu polarizacije mete. Za poprečno polarizirane mete očekujemo povećano odstupanje raspodjele u poprečnoj ravnini od aksijalne simetrije. Mislimo da ta poprečna nejednoličnost i konačno stanje pružaju jasno objašnjenje predznaka Siversovog doprinosa poluinkluzivnim jednospinskim asimetrijama.